**School of Mechatronic Systems Engineering**
**Simon Fraser University**
**MSE483/782 Midterm Exam**

February 23, 2017 (Duration: 2 hours)
**Please read the following before signing your name**

- The exam is closed-book. A 2-page formula sheet is permitted but must not contain any solved problems. The formula sheet has to be returned with the questions.
- Questions have an equal weight of 20% each. **Please clearly specify any assumptions you make and write legibly. You may lose marks if your work is not clear.**

Name: Student I.D. Number:

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1) Obtain a state-space representation of the following transfer function in the controllable canonical form

$$\frac{Y(s)}{U(s)} = \frac{100s}{s^2 + 100s + 9} + 1 \cdot$$

Show all steps in the derivation.

2) Linearize the system below about the origin and obtain the A, B, C, D state-space matrices.

$$
\begin{aligned}
\dot{x}_1 &= (-\alpha_1 + sin(x_2))x_1 + x_2 sin(x_2) + u \\
\dot{x}_2 &= x_1 sin(x_1) + x_2(-\alpha_2 + sin(x_2)) + u \\
y &= sin(x_1) + u cos(x_2)
\end{aligned}
$$

3) Convert the following state-space model into a diagonal representation

$$\begin{aligned}\dot{x} &= \begin{bmatrix} 1 & -4 \\ 3 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ -1 \end{bmatrix} u(t) \\ y &= [1 \quad 0] x \cdot\end{aligned}$$

4) Consider the state space system, $\dot{x} = Ax + Bu$, where $x \in R^n$ and $u \in R^m$. Assuming that $A$ has $n$ distinct eigenvalues, show that the zero-input response of the system can be written in the following form

$$x(t) = \sum_{i=1}^{n} \alpha_i e^{\lambda_i t} v_i$$

where $\lambda_i$ and $v_i$ are the $i - th$ eigenvalue and eigenvector of $A$, respectively, and $\alpha_i$'s are some real constants.

**Hint**: An arbitrary initial condition can be written as a linear combination of $n$ eigenvectors (if the eigenvectors form a basis, i.e., are linearly independent vectors).

5) Coupled tanks are common systems in process industries such as petro-chemical, pulp and paper, and water treatment. In these applications, liquids have to be pumped, mixed, stored, and transferred to other tanks. Thus control of the liquid levels is often required by regulating the liquid flows. Assuming that liquid flowing into and out of a tank are given by $Q_i$ and $Q_o$, respectively, the flow dynamics is given by $Q_i - Q_o = A\frac{dh}{dt}$; where $A$ is the cross sectional area of the tank, and h is the liquid level in the tank, respectively. If the valve is a sharp-edged orifice, its outflow rate is given by $Q_o = C_v a_v \sqrt{2gh}$; where $C_v$ is the discharge coefficient and $a_v$ is the cross sectional area of the valve, respectively; and $g$ is the gravity constant.
Using the above liquid balance relationship, obtain a state space representation for the system below. Investigate if the system is controllable at a given equilibrium point. If not, indicate how it can be re-designed to make it controllable.