Mechatronic Systems Engineering
Simon Fraser University
ENSC483/ENSC893 Midterm Exam

Date: February 21, 2013

Please read the following before signing your name

- You have 2 hours to write this examination.
- The exam is closed-book. You are allowed to bring a 2-page formula sheet.
- Questions are marked out of 100. Questions have to be returned with the answer book.
- Clearly specify any assumptions you make. Please write legibly. If your work is not clear, it may be marked as wrong.

Name: Student I.D. Number:

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1) (15 marks) The nodal equation for a parallel RLC circuit is given by

$$i_s(t) = C\frac{dv(t)}{dt} + i_L(t) + \frac{v(t)}{R}$$

where v is the voltage across the capacitor, $i_L(t)$ is the inductor current given by $i_L(t) = i_L(t_0) + \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau$, and $i_s(t)$ is the input forcing function, i.e., $u(t) = i_s(t)$. Obtain a state space representation of the circuit by defining appropriate state variables.

2) (15 marks) Obtain a state space representation of the single-input single-output system given by the transfer function

$$H(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0}\,.$$

3) Note: This question has 3 parts (a), (b), and (c) that can be solved independently.
For the state equation given by

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y &= [1 \quad 1]x \end{aligned}$$

(a) (15 marks) Find $y(t)$ when the input $u(t)$ is a unit step function and the initial state is zero at $t = 0$.
(b)(15 marks) Diagonalize the state equation.
(c) (15 marks) Obtain the transfer function from u to y.

4) (25 marks) Consider a double pendulum system on a moving cart as shown in the figure below. The dynamic equations are given by

$$J_i\ddot{\theta}_i - m_i g l_i \sin\theta_i + m_i l_i u cos\theta_i = 0, \quad i = 1,\ 2$$

where $J_1 = m_1 l_1^2,\ J_2 = m_2 l_2^2$ are the moments of inertia of the pendulums, $l_1,\ l_2$ are the lengths of the links assumed to have negligible masses, $m_1,\ m_2$ are the masses of the pendulums, and u is the acceleration input to the cart. Assess the controllability of the system around $\theta_1 = \theta_2 = 0,\ \dot{\theta}_1 = \dot{\theta}_2 = 0$ and obtain the condition under which the system becomes uncontrollable.