School of Mechatronic Systems Engineering
Simon Fraser University
Final Exam: Modern Control Systems (MSE483)

Date: April 20, 2017 (7pm, SUR 3310)

Please read the following before signing your name

- You have 2.5 hours to write this examination.
- The exam is closed-book. Calculators and a formula sheet are allowed.
- Questions are marked out of 100.
- Please write legibly and clearly specify any assumptions made. If your work is not clear, it may be marked as wrong.

Name: Student I.D. Number:

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1) (16 marks) Verify if the system given by

$$\begin{aligned}\dot{x} &= \begin{bmatrix}2 & 1 & 0\\ 0 & 2 & 0\\ 0 & 0 & -3\end{bmatrix} x + \begin{bmatrix}4 & 2\\ 0 & 0\\ 3 & 0\end{bmatrix} u\\ y &= \begin{bmatrix}0 & 0 & 1\end{bmatrix} x\end{aligned}$$

can be stabilized by the state feedback control $u = Kx$.

2) (16 marks) For the system given by

$$\begin{aligned}\dot{x} &= \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} x\end{aligned}$$

Indicate if we can find a control input u such that the final state $x_f = [4\ 1\ 4]^T$ can be reached starting from the initial state $x_i = [0\ 0\ 0]^T$. How about the final state $x_f^{'} = [1\ 1\ 1]^T$?

3) (16 marks) A state space representation of $G(s) = \frac{1}{s+1}$ is given by

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{aligned}$$

Obtain expressions for the states and output and comment on the stability of the system. Is the above state space realization suitable for simulating a system with transfer function $G(s)$?

4) (16 marks) Verify if the system given by the following dynamics is observable

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} x \end{aligned}$$

Next, let $u = [0\ 0\ 1]x$ and verify observability of the resulting closed-loop system. What general conclusion can you draw from this example?

5) (16 marks) The dynamics of a DC motor with an external load torque is described by

$$\begin{aligned} \dot{\theta} &= \omega \\ \dot{\omega} &= -\frac{1}{\tau}\omega + c_1 u + c_2 T_e \end{aligned}$$

where θ is the motor angle, ω is the angular speed, u the control input, T_e is the external load torque, and c_1, c_2, and τ are constants.

a) Consider the case when $T_e = 0$ and θ_r is constant. If the control law $u = l_0\theta_r - l_1\theta - l_2\omega$ is used, find l_0, l_1, l_2 such that the poles of the closed-loop system are $1 \pm j1$ and θ converges to θ_r in the steady-state.

b) Modify the above controller such that $\theta = \theta_r$ in the steady-state even for constant non-zero T_e and constant θ_r.

6) For the following system

$$\begin{aligned}\dot{x} &= \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x\end{aligned}$$

a) (10 marks) Suppose that we would like the output y to converge to a constant reference set point y_d. Obtain a state controller which can place the closed loop poles at -1 and -2, respectively, and achieve zero steady-state error.

b) (10 marks) Augment the original system dynamics with the equation $\dot{\xi} = y - y_d$ and design a state feedback controller $u = Kx - K_i\xi$ such that the poles of the closed loop system are placed at -1, -1, -2, respectively. Determine the closed-loop transfer function from y_d to y and verify that asymptotic convergence is achieved.

7) Consider a double integrator system given by the transfer function $Y(s)/U(s) = 1/s^2$. Assume that we use a PID controller given by $u = k_p e + k_d \dot{e} + k_i \int e$ to control the system so that the output y reaches a desired constant value given by y_r. Formulate the above PID control problem in the form of a state feedback controller and verify if the closed-loop eigenvalues can be placed arbitrarily in the left-half plane.