School of Mechatronic Systems Engineering
Faculty of Applied Sciences
Simon Fraser University
Final Exam: Modern Control Systems (ENSC483)

Date: April 18, 2013

Please read the following before signing your name

- You have 2.5 hours to write this examination.
- The exam is closed-book. Calculators and a formula sheet are allowed.
- Questions are marked out of 100.
- The question sheet has to be returned with the answer book.
- Please write legibly and clearly specify any assumptions made. If your work is not clear, it may be marked as wrong.

Name: Student I.D. Number:

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1) (17 marks) For the system characterized by

$$\begin{aligned} \dot{x} &= \begin{bmatrix} \mu & 1 & 0 \\ 0 & \mu & 1 \\ 0 & 0 & \mu \end{bmatrix} x + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} u \\ y &= \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} x \end{aligned}$$

obtain condition(s) for the system to be controllable and observable.

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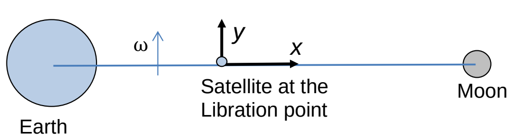
2) (17 marks) The dynamics of a satellite orbiting at the so called *libration point* between the moon and the earth are given by

$$\begin{aligned} \ddot{x} - 2\omega\dot{y} - 9\omega^2 x &= 0 \\ \ddot{y} + 2\omega\dot{x} + 4\omega^2 y &= u \end{aligned}$$

where x is a perturbation in the radial position, y is a perturbation in the azimuth position, u is the engine thrust in the y direction, m is the mass of the satellite, and $\omega = 2\pi/29$ (rad/day) is the angular speed of the moon relative to the earth.



a) With $u = 0$ show that the system is unstable at the libration point.
b) Design a state feedback controller to place the closed-loop system poles at $-3\omega,\ -4\omega,\ (-3 \pm 3j)\omega$.

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3) (18 marks) Many applications in industrial motor drives such as hoists, conveyors, and compressors involve handling constant load torques. The dynamic equations of a DC servomotor driving a constant load torque T_L are given by

$$\begin{aligned} L\frac{di}{dt} &= v - Ri - K_m\omega \\ J\frac{d\omega}{dt} &= K_m i + T_L \end{aligned}$$

where i is the motor current; v is the voltage applied to the motor terminals; ω is the motor angular speed; and L, R, K_m are the motor inductance, coil resistance, and torque constant, respectively. Obtain a mathematical scheme to estimate ω and T_L based on the measurement of motor current only.

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4) (a) (15 marks) Specify what is meant by the Separation Principle in state feedback control when using a full-order observer. State and prove it.

(b)(15 marks) Design a state feedback controller with a full-order observer for the following system such that the observer errors and closed-loop eigenvalues converge to zero at rates $exp(-50t)$ and $exp(-10t)$, respectively.

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & -1 \end{bmatrix} x \end{aligned}$$

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5) (18 marks) Two controllable and observable single-input single-output systems are cascaded in series, where the output of the first system is the input of the second system. Is the cascaded system both observable and controllable? Provide the necessary proof.
