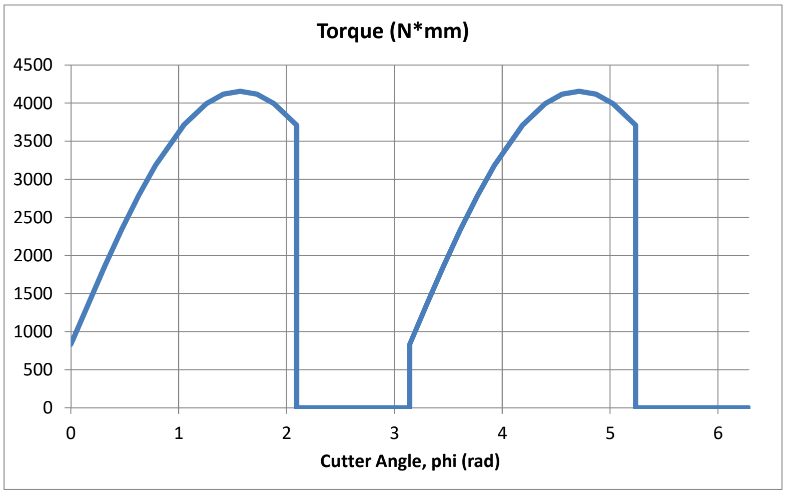
MSE 480/780 – Mid-Term Exam Solutions (Spring 2017)

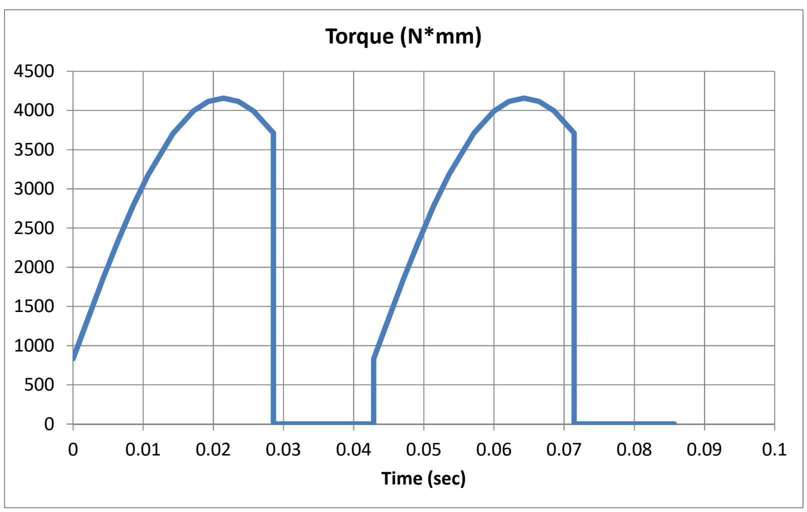
Problem 1 (20 Marks)

Part a (5 marks)

Plotting Tc versus cutter angle:



Alternatively plotting versus time:



Part b (5 marks)

- Max Cutting Torque, T_c: $RK_1as_t(\sin(\pi/2)+h^*/s_t)$ = **4.156 N*m**
- Max Tangential Force, F_t: $K_1as_t(\sin(\pi/2)+h^*/s_t)$ = **332.5 N**
- Max Thrust Force, F_{th}: $K_1as_t(r_1\sin(\pi/2)+r_2h^*/s_t)$ = **212.8 N**
- Max Radial Force, F_r: $F_{th}\cos(\Psi)$ = **150.5 N**
- Max Axial Force, F_z: $F_{th}\sin(\Psi)$ = **150.5 N**

Part c (5 marks)

Power can be calculated as:	P = (Torque)*(Spindle Speed)		
	=Omega*R*K1*a*st*(sin(phi) + h*/heq)		
Given 2/3 immersion and heq~st:	=> Pmax ~ Omega*R*K1*a*(st + h*)		
e.g. calculated (max) power from part a		304.7	W
Maximum spindle power		400	W
Maximum feed per tooth (st)max	=Pmax/(Omega*R*K1*a) - h*		
(st)max=		0.055645003	mm

Maximum feed per revolution (s = 2*st)	0.111	mm

**** Using Pmax is conservative; also OK to calculate based on Pavg as follows*

Integrating over 1/2 revolution (period of signal) with 2/3 immersion yields:

Pavg ~ (1/Pi)*Omega*R*K1*a*(1-cos((2*Pi)/3))*st + (2/3)*Omega*R*K1*a*h*

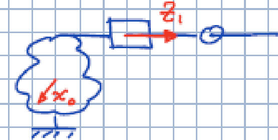
e.g. calculated (avg) power from part a		157.0	W
	Letting A =		
	(1/Pi)*Omega*R*K1*a*(1-cos((2*Pi)/3)) =	2909.4	
	and letting B =		
	(2/3)*Omega*R*K1*a*h* =	40.6	
	=> Pavg = A*st + B		
Maximum feed per tooth (st)max	=(Pmax - B) / A		
(st)max=		0.12352394	mm

Maximum feed per revolution (s = 2*st)	0.247	mm

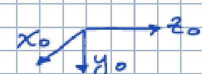
Problem 2 (20 Marks)

Part a (5 marks)

- LOOKING AT $A_{01}(\theta_1)$ IT CAN BE SEEN THAT JOINT #1 IS REVOLUTE.
- IT CAN ALSO BE SEEN THAT THE AXIS OF ACTUATION FOR JOINT #2 (z_1) IS ORIENTED AS SHOWN:

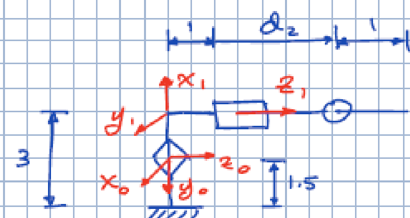


- THE ROTATION MATRIX R_{01} SHOWS THAT z_0 IS PARALLEL WITH z_1 SO FRAME F_0 IS ORIENTED AS FOLLOWS:



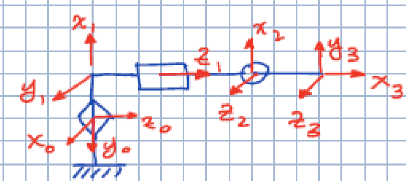
- WHEN $\theta_1 = 0$, THE DISPLACEMENT VECTOR $\vec{d}^{\,0}_{01} = [0, -1.5, 0]^T$ AND $R_{01} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow$



Part b (10 marks)

· ADDING FRAMES TO THE REST OF THE MANIPULATOR, FOLLOWING THE DH CONVENTION AND REFERRING TO $A_{01}(\theta_1)$ AS PROVIDED:



$\Rightarrow$

JOINT	a_i	α_i	d_i	θ_i
1	1.5	0	0	$\theta_1 - \pi/2$
2	0	$-\pi/2$	$1+d_2$	0
3	1	0	0	$\theta_3 - \pi/2$

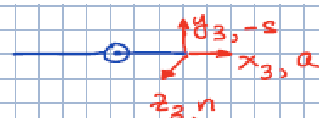
$\Rightarrow$

$$A_{12}(d_2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1+d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{23}(\theta_3) = \begin{bmatrix} s\theta_3 & c\theta_3 & 0 & s\theta_3 \\ -c\theta_3 & s\theta_3 & 0 & -c\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Part c (5 marks)

BY INSPECTION:



$$A_{3t} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$