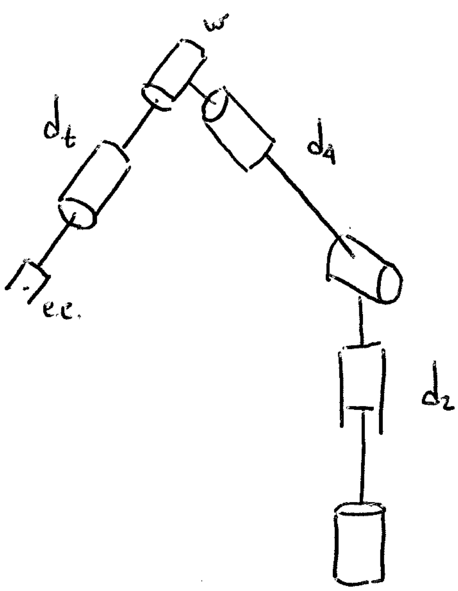
PREPARATION FOR TERM TEST II

For the manipulator shown below, determine ${}^{4}J_{\omega}$



$$ {}^{1}_{2}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad {}^{2}_{3}T = \begin{bmatrix} C_3 & -S_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S_3 & -C_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}_{4}T = \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ 0 & 0 & -1 & -d_4 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad {}^{4}_{5}T = \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S_5 & -C_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{5}_{6}T = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{4}T = \begin{bmatrix} C_3C_4 & -C_3S_4 & S_3 & S_3d_4 \\ S_4 & C_4 & 0 & 0 \\ -S_3C_4 & S_3S_4 & C_3 & C_3d_4+d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad {}^{4}_{6}T = \begin{bmatrix} C_5C_6 & -C_5S_6 & S_5 & 0 \\ S_6 & C_6 & 0 & 0 \\ -S_5C_6 & S_5S_6 & C_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$${}^4\hat{Z}_1 \times {}^4\bar{P}_{1\to w}$$

or

$$
{}^4J_w = \begin{bmatrix} {}^4\bar{P}_{w\to 1} \times {}^4\hat{Z}_1 & {}^4\hat{Z}_2 & {}^4\bar{P}_{w\to 3} \times {}^4\hat{Z}_3 & \bar{0} & \bar{0} & \bar{0} \\ {}^4\hat{Z}_1 & \bar{0} & {}^4\hat{Z}_3 & {}^4\hat{Z}_4 & {}^4\hat{Z}_5 & {}^4\hat{Z}_6 \end{bmatrix}
$$

In this case frame $\{4\}$ is located at the wrist

$$
{}^4\bar{P}_{w\to 3} = {}^4P_{4\to 3} = p\left({}^4_3T\right) = p\left({}^3_4T^{-1}\right) = -{}^3_4R^T\,{}^3\bar{P}_{3\to 4}
$$

$$
= \begin{bmatrix} C_4 & 0 & S_4 \\ -S_4 & 0 & C_4 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ d_4 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -d_4 \\ 0 \end{bmatrix}
$$

$$
{}^4\bar{P}_{w\to 1} = {}^4P_{4\to 1} = p\left({}^4_1T\right) = p\left({}^1_4T^{-1}\right) = -{}^1_4R^T\,{}^1\bar{P}_{1\to 4}
$$

$$
= \begin{bmatrix} C_3C_4 & S_4 & -S_3C_4 \\ -C_3S_4 & C_4 & S_3S_4 \\ S_3 & 0 & C_3 \end{bmatrix} \begin{bmatrix} S_3d_4 \\ 0 \\ C_3d_4 + d_2 \end{bmatrix} = \begin{bmatrix} S_3C_4d_2 \\ -S_3S_4d_2 \\ -d_4 - C_3d_2 \end{bmatrix}
$$

$$
{}^4\hat{Z}_4 = a(I) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix};\quad {}^4\hat{Z}_3 = a\left({}^3_4R^T\right) = \begin{bmatrix} S_4 \\ C_4 \\ 0 \end{bmatrix};\quad {}^4\hat{Z}_2 = a\left({}^2_4R^T\right) = \begin{bmatrix} -S_3C_4 \\ S_3S_4 \\ C_3 \end{bmatrix}
$$

$$
{}^4\hat{Z}_1 = a\left({}^1_4R^T\right) = \begin{bmatrix} -S_3C_4 \\ S_3S_4 \\ C_3 \end{bmatrix};\quad {}^4\hat{Z}_5 = a\left({}^4_5R\right) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix};\quad {}^4\hat{Z}_6 = a\left({}^4_6R\right) = \begin{bmatrix} S_5 \\ 0 \\ C_5 \end{bmatrix}
$$

$$
{}^4\bar{P}_{w\to 3} \times {}^4\hat{Z}_3 = \begin{bmatrix} 0 \\ 0 \\ -d_4 \end{bmatrix} \times \begin{bmatrix} S_4 \\ C_4 \\ 0 \end{bmatrix} = \begin{bmatrix} d_4C_4 \\ -d_4S_4 \\ 0 \end{bmatrix}
$$

$$
{}^4\bar{P}_{w\to 1} \times {}^4\hat{Z}_1 = \begin{bmatrix} S_3C_4d_2 \\ -S_3S_4d_2 \\ -d_4 - C_3d_2 \end{bmatrix} \times \begin{bmatrix} -S_3C_4 \\ S_3S_4 \\ C_3 \end{bmatrix} = \begin{bmatrix} S_3S_4d_4 \\ S_3C_4d_4 \\ 0 \end{bmatrix}
$$

$$
{}^{4}J_{w} = \begin{bmatrix}
S_3 S_4 d_4 & -S_3 C_4 & d_4 C_4 & 0 & 0 & 0 \\
S_3 C_4 d_4 & S_3 S_4 & -d_4 S_4 & 0 & 0 & 0 \\
0 & C_3 & 0 & 0 & 0 & 0 \\
-S_3 C_4 & 0 & S_4 & 0 & 0 & S_5 \\
S_3 S_4 & 0 & C_4 & 0 & 1 & 0 \\
C_3 & 0 & 0 & 1 & 0 & C_5
\end{bmatrix}
$$