Q1

A one-quadrant chopper is used to charge the battery as shown in Fig.1. The input dc voltage of the chopper is **150V**. The output of the chopper is connected to the battery through a **2.5mH** inductor. As shown in Fig.2, two charging stages are implemented by the controller, which adjusts the duty cycle of the chopper: 1) when the battery's voltage is lower than 24V, a **20A** constant charging current is applied to the battery; 2) when the battery's voltage is higher than 24V, a **1A** constant charging current is applied to charge the battery to full. The switching frequency of the chopper is **1000Hz**. Assume the internal impedance of the battery is zero and the battery's voltage is constant during each switching cycle.

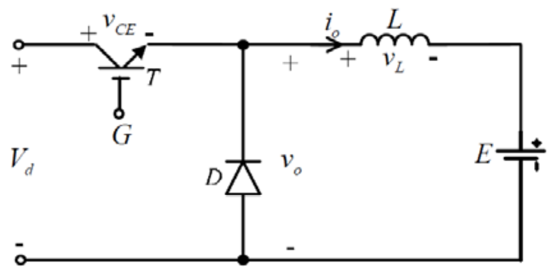


Figure 1 Battery charger

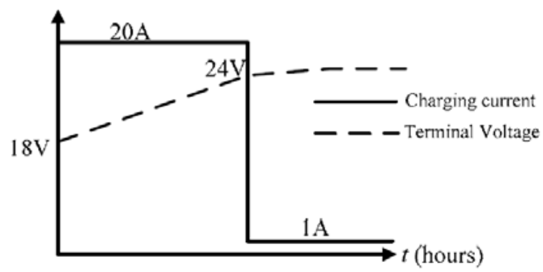


Figure 2 Two stages charging

At the beginning, the battery's voltage is **18V** and the average charge current is **20A**. Answer questions 1) and 2).

1) Determine the operation mode of the chopper.

Solution:

Assume it's in continuous current mode $\rightarrow V_o = DV_d \rightarrow D = V_o/V_d = {}^{18}\!/_{150} = 0.12$

$$\therefore I_{LB} = \frac{T_s V_d}{2L} D(1-D) = \frac{1\times 10^{-3}\times 150}{2\times 2.5\times 10^{-3}} \times 0.12\times(1-0.12) = 3.168\text{ A}$$

$\because I_L = I_o = 20\text{A} > I_{LB}$

$\therefore$ The charger is running in continuous current mode

2) Find the duty cycle, D.

Solution:

$\because$ It's running in continuous current mode

$\therefore V_o = DV_d \rightarrow$

$D = V_o/V_d = {}^{18}\!/_{150} = 0.12$

After few hours of charging, the battery's voltage reaches **24V** and a **1A** average current is applied to the battery. Answer questions 3) ~ 5)

3) Determine the operation mode of the chopper.

Solution:

Assume it's in continuous current mode$\rightarrow V_o = DV_d \rightarrow D = V_o/V_d = {}^{24}\!/_{150} = 0.16$

$$\therefore I_{LB} = \frac{T_s V_d}{2L} D(1-D) = \frac{1\times10^{-3}\times150}{2\times2.5\times10^{-3}} \times 0.16 \times (1-0.16) = 4.032\ \text{A}$$

$\because I_L = I_o = 1\text{A} < I_{LB}$

$\therefore$ The charger is running in discontinuous current mode

4) Find the duty cycle, D.

Solution:

It's running in discontinuous current mode

$$V_o = \frac{D^2}{D^2 + \frac{1}{4}\left(I_o/I_{LB,\max}\right)} V_d, \text{ where } I_{LB,\max} = \frac{T_s V_d}{8L} = \frac{1\times10^{-3}\times150}{8\times2.5\times10^{-3}} = 7.5\text{A}$$

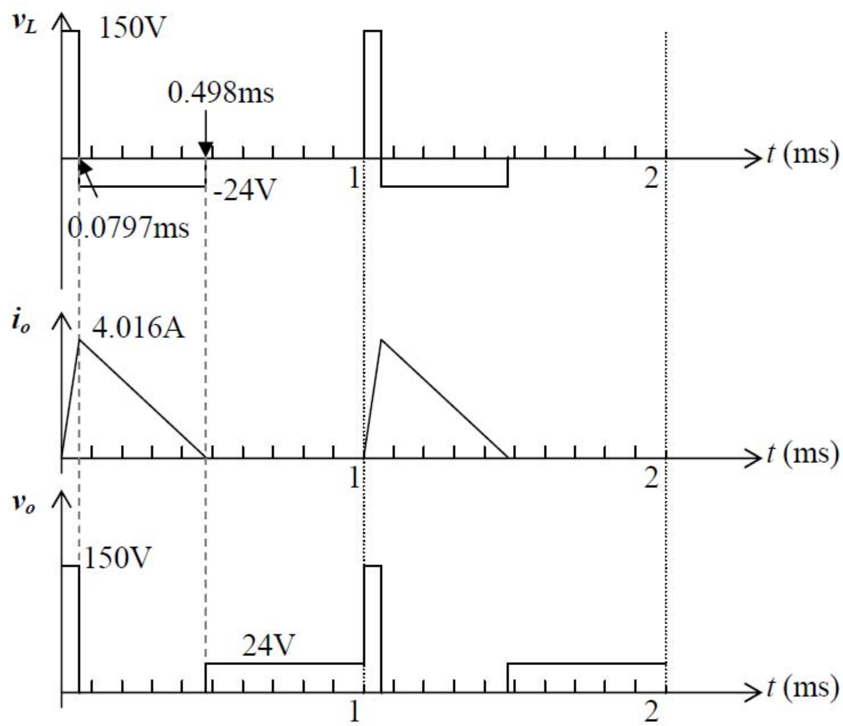
$$D = \sqrt{\frac{\frac{V_o}{4}\left(I_o/I_{LB,\max}\right)}{V_d - V_o}} = \sqrt{\frac{\frac{24}{4}\left({}^{1}\!/_{7.5}\right)}{150-24}} = 0.0797$$

5) Sketch to scale the following waveforms, indicating the peak values.

i) voltage waveform of the inductor, $\boldsymbol{v_L}$.

ii) output current waveform, $\boldsymbol{i_o}$

iii) output voltage waveform, $\boldsymbol{v_o}$



$$\Delta_1 = \frac{I_o}{4I_{LB,\max}D} = \frac{1}{4\times 7.5\times 0.0797} = 0.4183$$

$$I_{L,\max} = \frac{V_d - V_o}{L}t_{on} = \frac{150-24}{2.5\times 10^{-3}}\times 0.0797\times 10^{-3} = 4.016\text{ A}$$

Q2

a) The converter is runing in continuous current mode

$D = V_o/V_d = 5/15 = 0.333$

b). $\Delta I = \frac{V_d - V_o}{L} \cdot t_{on} = \frac{V_o}{L} \cdot t_{off}$

Method 1: $L = \frac{V_d - V_o}{\Delta I_L} \cdot \frac{D}{f_s} = 0.1667 mH$

Method 2: $L = \frac{V_o}{\Delta I_L} \cdot \frac{(1-D)}{f_s} = 0.1667 mH$

c). $\Delta V_o = \frac{\Delta I_L}{8} \cdot T_s/C \rightarrow C = \frac{\Delta I_L}{8 \Delta V_o \cdot f_s} = \frac{1}{8 \times 10 \times 10^{-3} \times 20 \times 10^3}$

$= 625 \mu H$

d) $I_{LB} = \frac{1}{2} i_{L,peak} = \frac{1}{2} \times 1 = 0.5 A$

$I_{OB} = I_{LB} = 0.5 A$

Q3

a) $V_o = 5V$, $R_L = 20\Omega$ $\rightarrow I_o = V_o/R_L = 0.25A$

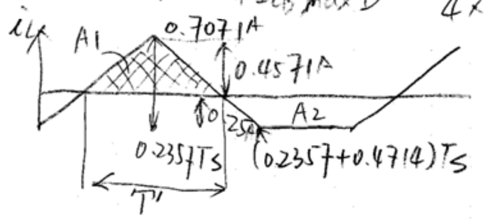
$I_o < I_{oB} \rightarrow$ The converter is running in DCM.

$$I_{LB,max} = \frac{T_s V_d}{8L} = \frac{V_d}{8 f_s L} = \frac{15}{8\times 20\times 10^3 \times 0.1667\times 10^{-3}} = 0.5625A$$

$$D = \sqrt{\frac{\frac{V_o}{4}(I_o/I_{LB max})}{V_d - V_o}} = \sqrt{\frac{\frac{5}{4}\times(0.25/0.5625)}{15-5}} = 0.2357$$

b) $$\Delta I_L = \frac{V_d - V_o}{L}\cdot t_{on} = \frac{15-5}{0.1667\times 10^{-3}} \times \frac{0.2357}{20\times 10^3} = 0.7071 A$$

c) $$\Delta_1 = \frac{I_o}{4 I_{LB max} D} = \frac{0.25}{4\times 0.5625 \times 0.2357} = 0.4714$$



$$\text{Area } A_1 = \frac{T' \times 0.4571A}{2}$$

$$\frac{T'}{(0.2357+0.4714)T_s} = \frac{0.4571}{0.7071}$$

$$T' = \frac{0.4571}{0.7071}\cdot(0.2357+0.4714)T_s = 22.85\mu s$$

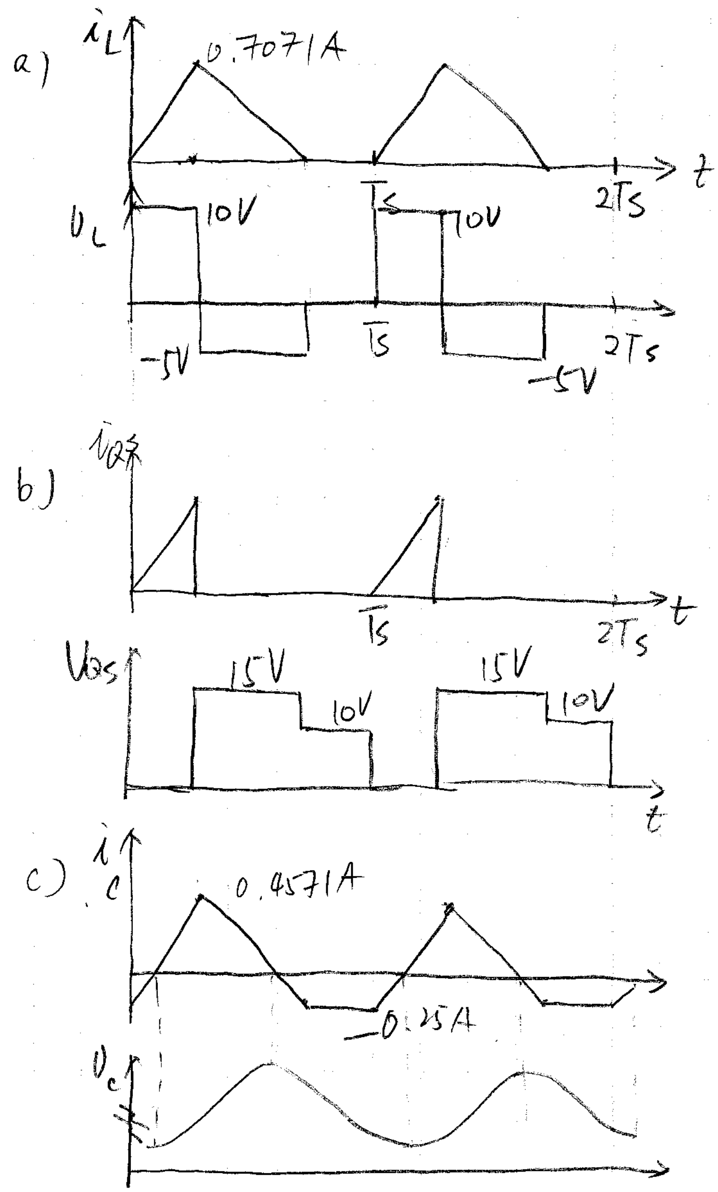
$$\Delta V_C = \frac{\text{Area } A_1}{C} = \frac{22.85\times 0.4571}{2C} = 8.4mV$$

$$\Delta V_o = \Delta V_C = 8.4mV$$

or $$\Delta V_C = \frac{\text{Area } A_2}{C} = \frac{(T_s - T') + (1 - D - \Delta_1)T_s}{2C}\times I_o = \frac{(50-22.85)+(1-0.2357-0.4714)\times 50}{2\times 625}\times 0.25$$

$$= 8.4mV$$

Q4


a)
i_L
0.7071A
t
Ts
2Ts
v_L
10V
10V
-5V
-5V
b)
i_Q
t
Ts
2Ts
V_{DS}
15V
10V
15V
10V
t
c)
i_C
0.4571A
-0.25A
v_C

Q5

a) Assume CCM $t_{on} = D/f_s = 0.8ms$ $\tau = L/R = 1.5ms$ $T_s = \frac{1}{f_s} = 2ms$

$$I_{min} = \frac{V_d}{R}\frac{e^{t_{on}/\tau}-1}{e^{T_s/\tau}-1} - \frac{E}{R} = -85.84A < 0 \quad \therefore DCM.\ I_{min} = 0$$

b) $$t_x = \tau \ln\left\{e^{t_{on}/\tau}\left[1+\frac{V_d-E}{E}(1-e^{-t_{on}/\tau})\right]\right\} = 0.0016s = 1.6ms$$

$$V_{a,avg} = \frac{t_{on}}{T_s}V_d + \frac{T_s-t_x}{T_s}E = 141.55V$$

$$I_{a,avg} = \frac{V_{a,avg}-E}{R} = \frac{141.55-110}{0.4} = 78.87A$$

c) $$I_{a,max} = \frac{V_d-E}{R}(1-R^{-t_{on}/\tau}) = 196.34A$$

d)