Q1. Figure 1 shows a magnetic core with two windings. The two windings ($N_1 = 150$, $N_2 = 150$) are connected in series and carry a current of 10A ($I_1 = I_2 = 10\text{A}$) as shown in Figure 1. The dimensions given in the figure are in millimeters. The length of each gap is **5mm**. The thickness of the core is **30mm**. The permeability of the core is assumed to be **infinite**. Neglect the fringing effect, answer questions 1.1 ~ 1.4

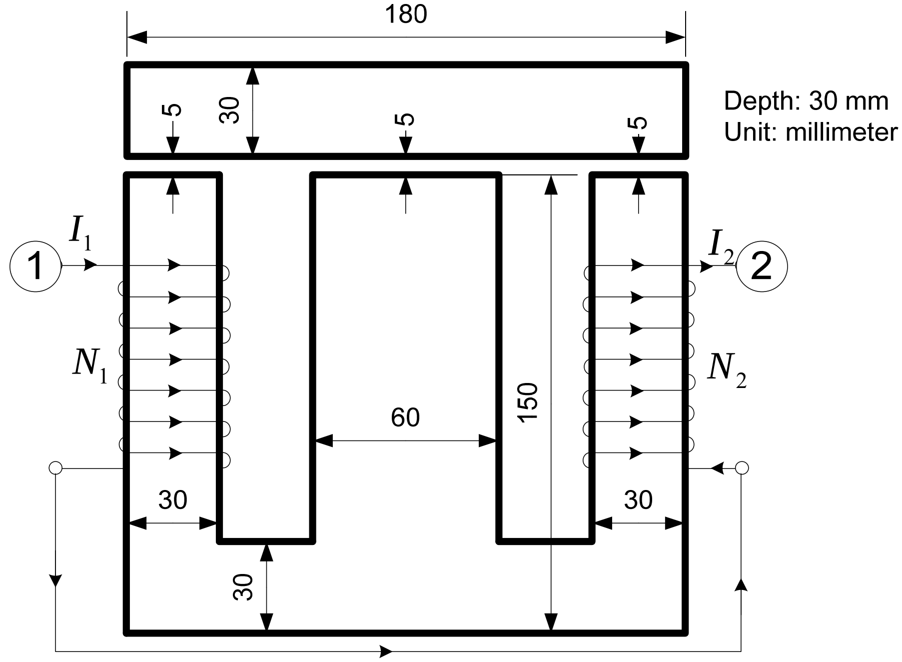
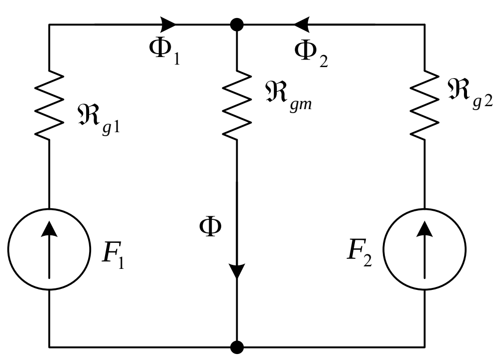


Figure 1 Magnetic core with two windings (unit: mm)

1.1) Draw the equivalent magnetic circuit. Eliminate the zero reluctance.

Solution:



$$\Re_{g1} = \Re_{g2} = \frac{l_g}{\mu_0 A} = \frac{5 \times 10^{-3}}{4\pi \times 10^{-7} \times (0.03 \times 0.03)} = 4.421 \times 10^6$$

$$\Re_{gm} = \frac{1}{2}\Re_{g1} = \frac{l_g}{\mu_0 A_m} = \frac{5 \times 10^{-3}}{4\pi \times 10^{-7} \times (0.03 \times 0.06)} = 2.2105 \times 10^6$$

$F_1 = F_2 = 150 \times 10 = 1500$ A·turns

1.2) Calculate the flux density B in the central air gap.

<u>Solution:</u>

$$F_1 = \Phi_1 \Re_{g1} + \Phi \Re_{gm}$$

$$F_2 = \Phi_2 \Re_{g2} + \Phi \Re_{gm}$$

$$\Phi = \Phi_1 + \Phi_2$$

$F_1 = F_2$ and $\Re_{g1} = \Re_{g2} = 2\Re_{gm}$, so $\Phi_1 = \Phi_2 = \dfrac{1}{2}\Phi$

$$\therefore \Phi = \frac{F_1}{2\Re_{gm}} = \frac{1500}{2 \times 2.2105 \times 10^6} = 3.393 \times 10^{-4}\ \text{Wb}$$

$$B = \frac{\Phi}{A_m} = \frac{3.393 \times 10^{-4}}{0.03 \times 0.06} = 0.1885\ \text{T}$$

1.3) Calculate the total equivalent inductance between node 1 and node 2.

$$\Phi_1 = \Phi_2 = \frac{1}{2}\Phi$$

$$L_{total} = \frac{N_1\Phi_1 + N_2\Phi_2}{i} = \frac{1}{2}\frac{(N_1 + N_2)\Phi}{i} = \frac{1}{2}\frac{(150 + 150) \times 3.393 \times 10^{-4}}{10} = 5.09\ \text{mH}$$

1.4) Calculate the mutual inductance between two windings.

$$\Re_{total} = \Re_{g1} + \Re_{gm} // \Re_{g2} = 2\Re_{gm} + \Re_{gm} // 2\Re_{gm} = 2\Re_{gm} + \frac{2}{3}\Re_{gm} = \frac{8}{3}\Re_{gm}$$

$$\Phi_1\big|_{F_2=0} = \frac{F_1}{\Re_{total}} = \frac{3F_1}{8\Re_{gm}}$$

$$\Phi_2\big|_{F_2=0} = \Phi_1\big|_{F_2=0} \frac{\Re_{gm}}{\Re_{gm} + \Re_{g2}} = \Phi_1\big|_{F_2=0} \frac{\Re_{gm}}{\Re_{gm} + 2\Re_{gm}} = \frac{1}{3}\Phi_1\big|_{F_2=0} = \frac{1}{3}\frac{3F_1}{8\Re_{gm}} = \frac{F_1}{8\Re_{gm}}$$

$$M = \frac{N_2\Phi_2\big|_{F_2=0}}{i_1} = \frac{N_2}{i_1}\frac{F_1}{8\Re_{gm}} = \frac{N_2 N_1 i_1}{i_1 \times 8\Re_{gm}} = \frac{N_2 N_1}{8\Re_{gm}} = \frac{150 \times 150}{8 \times 2.2105 \times 10^6} = 1.27\ \text{mH}$$

$$L_1 = L_2 = \frac{N_1^2}{\Re_{total}} = \frac{3N_1^2}{8\Re_{gm}} = \frac{3 \times 150^2}{8 \times 2.2105 \times 10^6} = 3.82\ \text{mH}$$

$$\therefore L_{total} = L_1 + L_2 - 2M$$

Q2.

Test results from a single phase, 10KVA, 2200V/220V, 60Hz transformer are as follows:

	Terminal voltage, V_t (V)	Terminal Current, I_t (A)	Input power, P_{in} (W)
Open circuit test (**LV side**)	220	2.5	100
Short circuit test (**HV side**)	150	4.55	215

2.1) Determine the parameters (R_{eq} , X_{eq} , R_c, and X_m) of the transformer referred to the **HV** side

$$R_{c,LV} = \frac{V_t^2}{P} = \frac{220^2}{100} = 484\,\Omega$$

$$I_c = \frac{V_t}{R_{c,LV}} = \frac{220}{484} = 0.4545\text{ A}$$

$$I_m = \sqrt{I_t^2 - I_c^2} = \sqrt{2.5^2 - 0.4545^2} = 2.4583\text{ A}$$

$$X_{m,LV} = \frac{V_t}{I_m} = \frac{220}{2.4583} = 89.49\ \Omega$$

referred to the HV side,

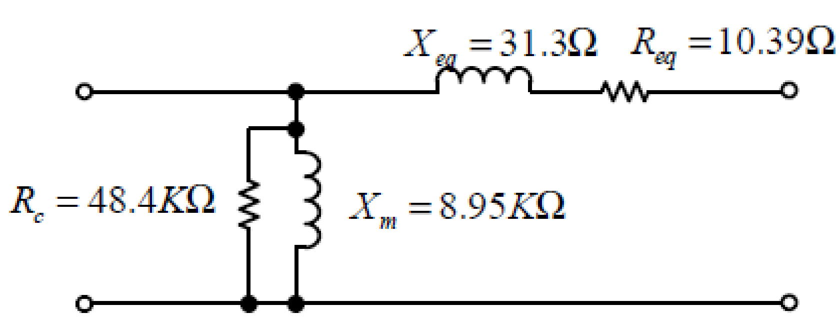
$$R_c = a^2 R_{c,LV} = \left(\frac{2200}{220}\right)^2 484 = 48.4\ \text{K}\Omega$$

$$X_m = a^2 X_{m,LV} = 10^2 \times 89.49 = 8.949\ \text{K}\Omega$$

$$R_{eq} = \frac{P}{I_t^2} = \frac{215}{4.55^2} = 10.3852\,\Omega$$

$$X_{eq} = \sqrt{\left(\frac{V_t}{I_t}\right)^2 - (R_{eq})^2} = \sqrt{\left(\frac{150}{4.55}\right)^2 - 10.3852^2} = 31.2885\,\Omega$$

2.2) Draw the approximate equivalent circuit of the transformer referred to the **HV** side



Q3. A shunt dc motor has the rated armature current of 30A. The total resistance in the armature circuit is 0.5Ω. The field winding resistance is 100Ω. When the shunt motor is connected to a 180V dc power supply, the terminal current is 5A and the motor rotates at 1800rpm without any mechanical load. Assuming that the terminal voltage remains at 180V and the rotational loss is constant, answer questions 3.1~3.5.

3.1) Calculate the rotational loss.

$$I_a = I_t - I_f = I_t - \frac{V_t}{R_f} = 5 - \frac{180}{100} = 3.2 \text{ A}$$

$$E_a = V_t - I_a R_a = 180 - 3.2 \times 0.5 = 178.4 \text{ V}$$

$$P_{rot} = E_a I_a = 178.4 \times 3.2 = 570.9 \text{ W}$$

3.2) Calculate the developed torque and motor speed at the rated armature current, assuming no armature reaction

$$I_{a,rated} = 30 \text{ A}$$

at no load, $K_a \Phi_{NL} = \frac{E_a}{\omega_m} = \frac{178.4}{\frac{1800}{60} \times 2\pi} = 0.946 \text{ V/rad/s}$

$$K_a \Phi_{FL} = K_a \Phi_{NL} = 0.946 \text{ due to no AR}$$

$$T = K_a \Phi_{FL} I_{a,rated} = 0.946 \times 30 = 28.39 \text{ Nm}$$

$$E_{a,FL} = V_t - I_a R_a = 180 - 30 \times 0.5 = 165 \text{ V}$$

$$\omega_{m,FL} = \frac{E_{a,FL}}{K_a \Phi_{FL}} = \frac{165}{0.946} = 174.34 \text{ rad/s}$$

$$n = \frac{\omega_{m,FL}}{2\pi} \times 60 = \frac{174.34}{2\pi} \times 60 = 1665 \text{ rpm}$$

3.3) Calculate the efficiency at the rated armature current, assuming no armature reaction.

$$P_{out} = E_{a,FL} I_{a,rated} - P_{rot} = 165 \times 30 - 570.9 = 4379.1 \text{ W}$$

$$P_{in} = V_t I_t = V_t (I_a + I_f) = 180 \times \left(30 + \frac{180}{100}\right) = 5724 \text{ W}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{4379.1}{5724} = 76.5\%$$

3.4) Calculate the developed torque at the rated armature current, assuming 10% reduction of the flux due to armature reaction

$$K_a \Phi_{FL} = 90\% K_a \Phi_{NL} = 0.9 \times 0.946 = 0.852 \text{ due to AR}$$

$$T = K_a \Phi_{FL} I_{a,rated} = 0.852 \times 30 = 25.55 \text{Nm}$$

3.5) An external rheostat is connected in series with the field winding to maintain the speed at 1800rpm. Calculate the resistance of rheostat when the motor is running with the rated armature current, assuming the flux is proportional to field current and no armature reaction

$$K_a\Phi_{FL} = K_a\Phi_{NL} = 0.946 \text{ due to no AR}$$

$$\omega_m = \frac{n}{60}\times 2\pi = \frac{1800}{60}\times 2\pi = 188.5\,\text{rad/s}$$

$$E_a = V_t - I_a R_a = 180 - 30\times 0.5 = 165\text{ V}$$

$$K_a\Phi_{new} = \frac{E_a}{\omega_m} = \frac{165}{188.5} = 0.8754\,\text{A}$$

$$\because \frac{K_a\Phi_{new}}{K_a\Phi_{NL}} = \frac{I_{f,new}}{I_f}$$

$$\therefore I_{f,new} = I_f\,\frac{K_a\Phi_{new}}{K_a\Phi_{NL}} = \frac{180}{100}\,\frac{0.8754}{0.946} = 1.665\,\text{A}$$

$$R_{fc} = \frac{V_t}{I_{f,new}} - R_{fw} = \frac{180}{1.665} - 100 = 8.12\Omega$$

Q4.

A 220V, 50A, 1500 rpm separately excited DC motor is supplied by its rated voltage. The motor is running at full load with 50A armature current. The armature resistance is 0.2Ω. The rotational loss is 600W at 1500rpm. The field current is maintained constant by a separate dc power supply. Assume that the armature reaction at full load causes 10% flux reduction. Answer following questions.

4.1) Find the load torque when the motor is running with 50A armature current.

4.2) Find the no load speed without considering the rotational loss

4.1) @ Full load, $I_a = 50A$, $V_t = 220V$ and $n = 1500 rpm$

$$E_a = V_t - I_a R_a = 220 - 50 \times 0.2 = 210V$$

$$P_{out} = E_a I_a - P_{rot} = 210 \times 50 - 600 = 9900 W$$

$$T_L = \frac{P_{out}}{\omega_m} = \frac{9900}{\frac{1500}{60} \times 2\pi} = 63 \ Nm$$

(157.08)

or: $$K_a\phi_{FL} = \frac{E_a}{\omega_m} = \frac{210}{\frac{1500}{60} \times 2\pi} = 1.3369 \ V\ sec/rad$$

$$T = K_a\phi_{FL} I_a = 1.3369 \times 50 = 66.85 \ Nm$$

$$T_L = T - \frac{P_{rot}}{\omega_m} = 66.85 - \frac{600}{157.08} = 63 \ Nm$$

(3.8197)

4.2) $$K_a\phi_{NL} = \frac{K_a\phi_{FL}}{0.9} = \frac{1.3369}{0.9} = 1.4854$$

Without considering P_{rot}, $I_a \approx 0$. $E_a \approx V_t = 220V$.

$$\omega_m = \frac{E_a}{K_a\phi_{NL}} = \frac{220}{1.4854} = 148.1 \ rad/sec$$

$$n_m = 1414.3 \ rpm$$

or $P_{rot} = 600V \rightarrow$ $600 = (V_t - R_a I_a) I_a \Rightarrow I_a = 2.7341A$

$E_a = 219.45 V$ $\omega_m = 147.73, \rightarrow n = 1410.8 \ rpm$

Q5.

A 100hp, 3-phase, 60Hz, 4-pole induction motor is running at 1750 rpm. At this speed it is generating output power of 60kW. Its stator copper loss is 2.2kW. Its estimated core and the rotational losses are 2kW and 1.5kW, respectively. Calculate its efficiency at 1750 rpm.

$P_{rot} = 1.5 KW$, $P_{core} = 2 KW$ $P_{cu1} = 2.2 KW$

$P_{out} = 60 KW$ $n = 1750$ rpm

$$n_s = \frac{120}{P} f_1 = \frac{120}{4} \times 60 = 1800 \text{ rpm}$$

$$S = \frac{n_s - n}{n_s} = \frac{1800 - 1750}{1800} = 0.0278$$

$$P_m = P_{out} + P_{rot} = 60 + 1.5 = 61.5 \text{ KW}$$

$$P_{ag} = \frac{P_m}{1-S} = 63.26 \text{ KW}$$

$$P_{in} = P_{ag} + P_{core} + P_{cu1} = 63.26 + 2.2 + 2 = 67.46 \text{ KW}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{60}{67.46} = 88.93\%$$