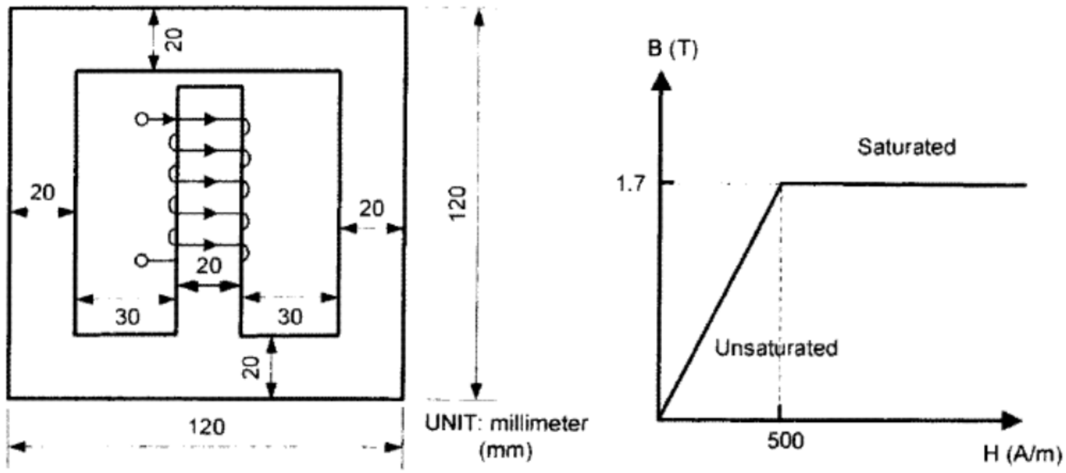
Q1. Fig. 1 shows the structure of an inductor. The unit of the dimensions is millimeter (**mm**). The B-H curve of the core is also shown, which is simplified from the real B-H curve with two asymptotes. The thickness of the core is **20 mm**. The length of the air gap is **1 mm**. The **100 turns** winding is mounted on the center leg. Neglect the fringing effect and winding resistance. If the excitation voltage (voltage across the winding) is **10 V** (AC rms, 60 Hz), determine the rms value of the excitation current.



Q2. Test results from a single phase, **20 kVA**, **4160 V/208 V**, **60 Hz** transformer are as follows:

	Terminal voltage, V_t (V)	Terminal Current, I_t (A)	Input power, P_{in} (W)
Open circuit test (**LV side**)	208	3.2	73
Short circuit test (**HV side**)	153	2.6	54

a) Determine the parameters (Req , Xeq , Rc, and Xm) of the transformer referred to the HV side.
b) Draw the approximate equivalent circuit of the transformer referred to the HV side.

Q3. A shunt dc motor has the rated armature current of **40 A**. The total resistance in the armature circuit is **0.3 Ω** and the total resistance in the field circuit is **100 Ω**. When the shunt motor is connected to a **180 V** power supply, the terminal current is **4.8 A** and the motor rotates at **1500 rpm** without any mechanical load. Assuming that the terminal voltage remains at **180V** and the rotational loss is constant, answer the following questions:

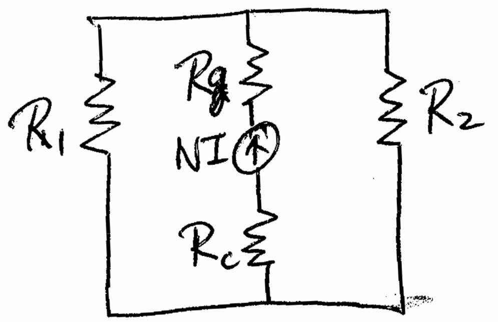
a) Calculate rotational loss P_{rot},
b) Calculate speed and torque at the rated armature current, assuming no armature reaction,
c) Calculate the efficiency at the rated armature current, assuming no armature reaction,

d) Calculate the speed, torque at the rated armature current assuming a 6% flux reduction due to armature reaction.

Q4. A **440 V**, **60 Hz**, Y-connected, 4-pole slip ring (wound rotor) induction motor has the following circuit parameters: $R_1 = 0.6\Omega$, $R_2^{'} = 0.5\Omega$, $X_1 = X_2^{'} = 0.8\Omega$, $X_m = 50\Omega$, calculate the following:

a) The starting line current (stator current) I_1,
b) Starting torque T_{st},
c) Speed of rotation n in rpm when the shaft developed torque is **160 Nm**
d) Maximum developed torque, T_{max}

Q1. If $B_c \times 1.7T$. $\mu_c = 1.7/500 = 0.0034$ $A = 20 \times 20 = 400 mm^2$



$$R_g = \frac{l_g}{\mu_0 A} = \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times (0.02 \times 0.02)}$$

$$= 1.9894 \times 10^{6}$$

$$R_c = \frac{l_c}{\mu_c A} = 7.3529 \times 10^{4}$$

$$R_1 = R_2 = 1.4705 \times 10^{5}$$

$$V = E = 10V = 4.44 f N \phi_{PK} \rightarrow \phi_{PK} = \frac{10}{4.44 \times 60 \times 100}$$

(no resistance)

$$= 3.7538 \times 10^{-4} \text{ wb}$$

$$B_{pk} = \frac{\phi_{PK}}{A} = 0.9384 T < 1.7T$$

$$NI_{pk} = \phi_{PK} \cdot R_+ \Rightarrow I = \frac{\phi_{PK} \cdot R_+}{N \cdot \sqrt{2}} = \frac{3.7538 \times (R_1 // R_2 + R_c + R_g)}{100 \times \sqrt{2}}$$

$$= 5.6709 A$$

Q2 a) $R_{eq} = 7.988\Omega$ $X_{eq} = 58.3\Omega$ $R_c = 237.06k\Omega$ $X_m = 26.16k\Omega$

$$R_{c,LV} = \frac{V_t^2}{P} = \frac{208^2}{73} = 592.66\Omega$$

$$I_c = V_t / R_{c,LV} = 208/592.66 = 0.35096A$$

$$I_m = \sqrt{I_t^2 - I_c^2} = \sqrt{3.2^2 - 0.35096^2} = \sqrt{10.1168} = 3.1807A$$

$$X_{m,LV} = V_t / I_m = 208/3.1807 = 65.39\Omega$$

referred to the HV side :

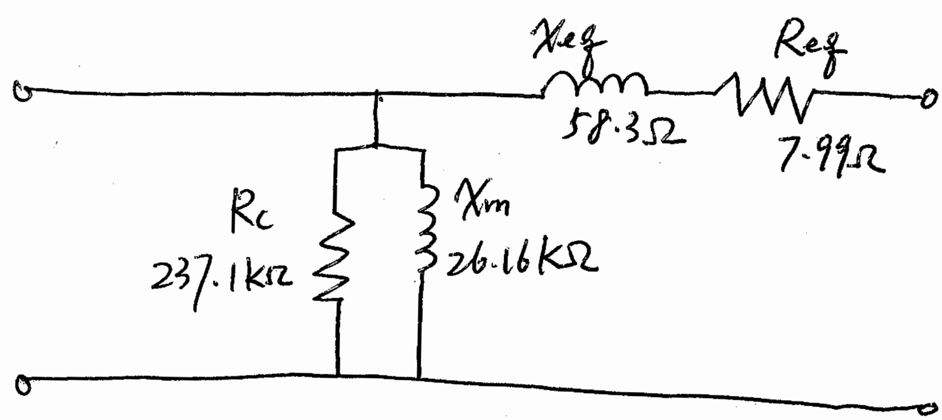
$$R_c = a^2 R_{c,LV} = \left(\frac{4160}{208}\right)^2 592.66 = 237.064\,k\Omega$$

$$X_m = a^2 X_{m,LV} = 20^2 \times 65.39 = 26.16k\Omega$$

$$R_{eq} = P/I_t^2 = 54/2.6^2 = 7.9882\Omega$$

$$X_{eq} = \sqrt{(V_t/I_t)^2 - (R_{eq})^2} = \sqrt{\left(\frac{153}{2.6}\right)^2 - 7.9882^2} = \sqrt{3399.06} = 58.3\Omega$$

b)



Q3. a) $P_{rot} = 537.3\,W$

$$I_f = V_t / R_f = 180/100 = 1.8A, \quad I_a = I_t - I_f = 4.8 - 1.8 = 3A$$

$$P_{ele} = E_{a,NL} I_{a,NL} = (V_t - I_a R_a) I_a = (180 - 0.3\times3)\times 3 = 537.3W$$

$$P_{rot} = P_{ele} = 537.3\,W$$

b) $n = 1407.26$ rpm $\quad T = 45.6$ Nm

$$E_{a,NL} = V_t - I_a R_a = 180 - 0.3\times 3 = 179.1\,V$$

$$\omega_{NL} = \frac{1500}{60} \times 2\pi = 157.08 \text{ rad/s}$$

$$K_a\phi = E_{a,NL} / \omega_{NL} = 179.1/157.08 = 1.14\,V\cdot\text{sec/rad}$$

$$E_{a,FL} = V_t - I_a R_a = 180 - 0.3\times 40 = 168\,V$$

$$\omega_{FL} = E_{a,FL} / K_a\phi = 168/1.14 = 147.368 \text{ rad/s}$$

$$n = \frac{\omega_{FL}}{2\pi} \times 60 = \frac{147.386}{2\pi} \times 60 = 1407.26 \text{ rpm}$$

$$T = K_a\phi I_a = 1.14 \times 40 = 45.6\,Nm$$

c) $\eta = 0.822$

$$P_{in} = V_t(I_f + I_a) = 180 \times (40 + \frac{180}{100}) = 7524W$$

$$P_{ele} = E_{a,FL} I_{a,FL} = 168 \times 40 = 6720W$$

$$P_{out} = P_{ele} - P_{rot} = 6720 - 537.3 = 6182.7\,W$$

$$\eta = P_{out}/P_{in} = 6182.7/7524 = 0.822$$

d) $n = 1497.1$ rpm $\quad T = 42.864$ Nm

$$K_a\phi_{FL} = 0.94 K_a\phi_{NL} = 0.94 \times 1.14 = 1.0716 \text{ V·sec/rad}$$

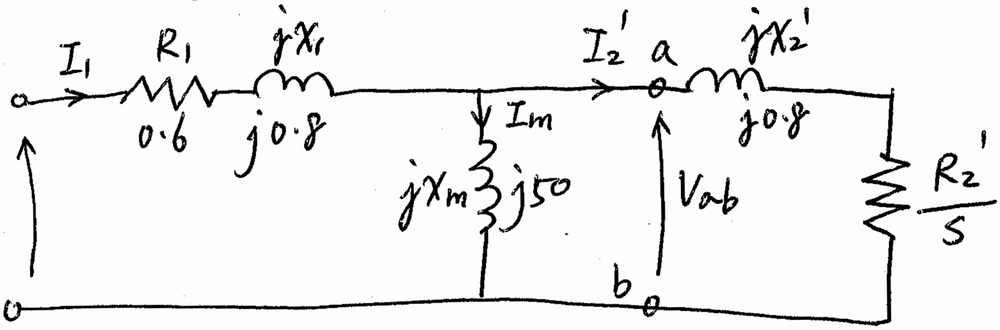
$$E_{a,FL} = V_t - I_a R_a = 180 - 0.3 \times 40 = 168 \text{ V}$$

$$\omega_{FL} = \frac{E_{a,FL}}{K_a\phi_{FL}} = \frac{168}{1.0716} = 156.775 \text{ rad/s}$$

$$n = \frac{\omega_{FL}}{2\pi} \times 60 = \frac{156.775}{2\pi} \times 60 = 1497.1 \text{ rpm}$$

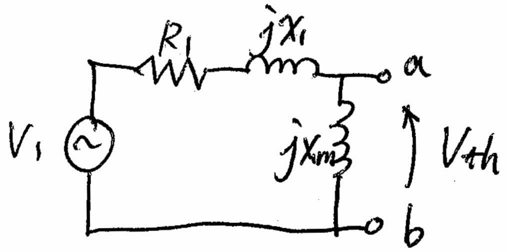
$$T = K_a\phi_{FL} I_a = 1.0716 \times 40 = 42.864 \text{ Nm}$$

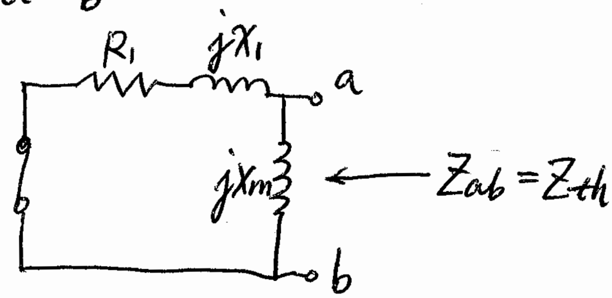
Q4. IEEE equivalent circuit of the motor



$V_1 = \frac{V_{LL}}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.04V$

Thevenize the circuit at a–b

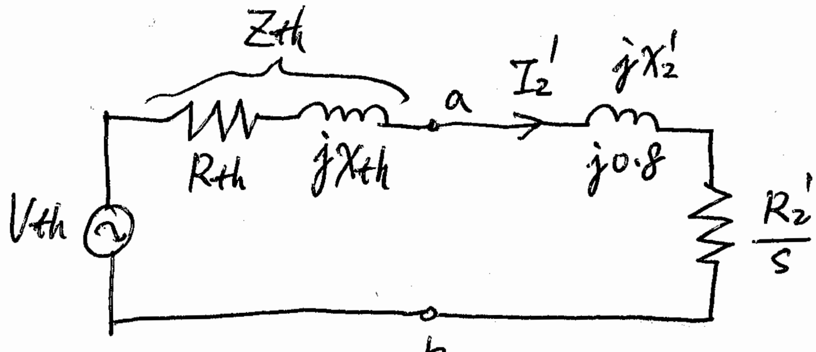




$$V_{th} = V_{ab} = \frac{jX_m}{R_1 + j(X_1 + X_m)} V_1 = \frac{j50}{0.6 + j50.8} \cdot V_1 = 0.984 \angle 0.7^\circ . V_1$$

$$V_1 = 254.04V \Rightarrow V_{th} = 0.984\angle 0.7^\circ (254.04 \angle 0^\circ) = 249.98\angle 0.7^\circ$$

$$Z_{th} = (R_1 + jX_1) // jX_m = \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)} = \frac{j50(0.6 + j0.8)}{0.6 + j50.8}$$



$$= 0.984 \angle 53.83^\circ$$

$$= 0.581 + j0.794 \Omega$$

a) Z_t at any speed or slip is $Z_t = Z_{th} + jX_2' + \frac{R_2'}{S} = 0.581 + j0.794 + j0.8 + \frac{0.}{s}$

when starting, $n_r = 0. \Rightarrow S = \frac{n_s - n_r}{n_s} = \frac{n_s - 0}{n_s} = 1$

$\therefore Z_T = 0.581 + j0.794 + j0.8 + \frac{0.5}{1} = 1.081 + j1.594 = 1.926\angle 55.86^\circ (\Omega)$

$I_2' = \frac{V_{th}}{Z_T} = \frac{249.98\angle 0.7^\circ}{1.926\angle 55.86^\circ} = 129.792\angle -55.16^\circ (A)$

$\therefore V_{ab} = I_2' \left(\frac{R_2'}{1} + jX_2' \right) = 129.792\angle -55.16^\circ (0.5 + j0.8)$

$= 129.792\angle -55.16^\circ (0.9434\angle 57.995^\circ)$

$= 122.45\angle 2.84^\circ (V)$

now go back to IEEE equivalent circuit to calcute I_m and I_1

$I_m = V_{ab}/jX_m = \frac{122.45\angle 2.84^\circ}{50\angle 90^\circ} = 2.449\angle -87.16^\circ (A)$

$I_1 = I_2' + I_m = 129.792\angle -55.16^\circ + 2.449\angle -87.16^\circ$

$= 131.881\angle -55.73^\circ (A)$

b) $T = 3 \frac{(I_2')^2 \frac{R_2'}{s}}{\omega_s}$, at starting, $s = 1$

$n_s = \frac{120f}{p} = \frac{120(60)}{4} = 1800 \text{ rpm}$ $\quad \omega_s = \frac{1800}{60} \cdot 2\pi = 188.5 \text{ rad/sec}$

$T_{st} = 3 \cdot \frac{(129.792)^2 \left(\frac{0.5}{1}\right)}{188.5} = 134.053 \text{ Nm}$

c) Torque at any slip $T = \frac{3}{\omega_s}(I_2')^2 \frac{R_2'}{s}$

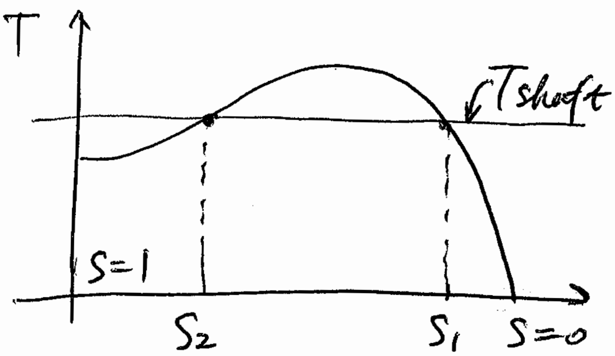
From the thevenin equivalent circuit

$$|I_2'| = \frac{|V_{th}|}{\sqrt{(R_{th} + \frac{R_2'}{s})^2 + (X_{th} + X_2')^2}} \Rightarrow |I_2'|^2 = \frac{(249.98)^2}{(0.581 + \frac{0.5}{s})^2 + (1.594)^2}$$

$$T = 160 = \frac{3}{188.5} \cdot \frac{(249.98)^2}{(0.581 + \frac{0.5}{s})^2 + (1.594)^2} \cdot \left(\frac{0.5}{s}\right)$$

$$160 = \frac{497.268 \cdot s}{2.879 s^2 + 0.531 s + 0.25} \Rightarrow 2.879 s^2 - 2.577 s + 0.25 = 0$$

$$\Rightarrow S_1 = 0.111 \; \& \; S_2 = 0.785$$



S_2 operates in unstable region, only S_1 is acceptable

$\therefore n = (1 - S_1) n_s$

$= (1 - 0.111) 1800 = 1600.2$ rpm

d) at T_{max}. $\frac{R_2'}{S_m} = \sqrt{R_{th}^2 + (X_{th} + X_2')^2}$

$$\Rightarrow S_m = \frac{R_2'}{\sqrt{R_{th}^2 + (X_{th} + X_2')^2}}$$

$$= \frac{0.5}{\sqrt{(0.581)^2 + (0.794 + 0.8)^2}} = 0.295$$

Calculate I_2' using Thevenin circuit

$$I_2' = \frac{V_1}{\left(R_{th} + \frac{R_2'}{S_m}\right) + j\left(X_{th} + X_2'\right)} = \frac{249.98\angle 0.7^\circ}{\left(0.581 + \frac{0.5}{0.295}\right) + j\left(0.794 + 0.8\right)}$$

$$= 89.953\angle 34.31^\circ \text{ (A)}$$

$$\therefore T_{max} = \frac{3}{\omega_s}\left(I_2'\right)^2 \frac{R_2'}{S_m} = \frac{3}{188.5}\left(89.953\right)^2\left(\frac{0.5}{0.295}\right) = 218.27 \text{ N.m}$$