Simon Fraser University

School of Mechatronic Systems Engineering

MSE280 Linear Systems, Midterm exam

Fenruary 23 (Thursday), 2017, 9:30am-11:20am

Name: Student #:

Exam policies:

Solutions

- Closed-book.
- One page letter-size hand-written cheat-sheet (both sides) is allowed.
- Write all your answers on this booklet. No extra sheet will be provided.
- Motivate your answers properly.
- Mark may be scaled later.
- Turn off your mobile phone and/or smart watch.

To be filled in by the instructor/marker

Problem #	Mark	Full Mark
1		6
2		6
3		6
4		6
5		6
Total		30

1. Select one correct statement (by circling one of the numbers i, ii, iii, or iv for the following sentences. **Justify your answers**. [1.5 points each]

1.1 A CT LTI system has impulse response

$$h(t) = e^{(ct)}u(t) \qquad 0 < c < 1$$

where $u(t)$ is unit step function. Then the system is:

i) memoryless

ii) causal

iii) stable

iv) memoryless & causal

a. h(t) only depends on U(t), that is the output depends only on the input at the same time.

b. also the system is causal as it is memoryless.

1.2 What is the fundamental period of the signal

$$x[n] = e^{j\frac{7}{6}\pi n}$$

i) 24

ii) 6

iii) 7

iv) 12

$x[n] = x[n+N] \Rightarrow e^{j\frac{7}{6}\pi n} = e^{j\frac{7}{6}\pi(N+n)} \Rightarrow 1 = e^{j\frac{7}{6}\pi N}$

$\Rightarrow e^{j2\pi m} = e^{j\frac{7}{6}\pi N} \Rightarrow 2\cancel{\pi}m = \frac{7}{6}\cancel{\pi}N \Rightarrow N = \frac{12}{7}m \Rightarrow$ if $m = 7 \Rightarrow \boxed{N = 12}$

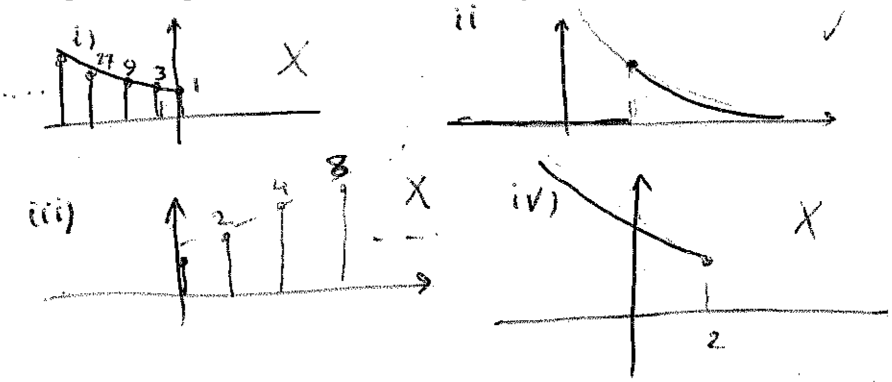
1.3 Which of the following is an impulse response for a <u>**stable**</u> LTI system?

i) $h[n] = (\frac{1}{3})^n u[-n]$

ii) $h(t) = \frac{1}{t^2}u(t-2)$

iii) $h[n] = 2^n u[n]$

iv) $h(t) = e^{-t}u(-t+2)$



1.4 Consider a DT system whose input $x[n]$ and output $y[n]$ are related by:

$$y[n] = (\tfrac{1}{2})y[n-1] + x[n].$$

If the system satisfies the condition of initial rest at $n < 0$ (i.e. if x[n]=0 for $n < 0$, then $y[n] = 0$ for $n < 0$), then:

i) it is linear and time invariant.

ii) it is linear and time variant.

iii)it is nonlinear and time invariant.

iv)it is nonlinear and time variant.

if the system is in intial rest at $n<0 \Rightarrow$ if $x[n]=0$ for $n<0$ $\Rightarrow y[n]=0$ $n<0$

$\Rightarrow y[-1]=0 \Rightarrow$

$y[0] = \frac{1}{2} y[-1] + x[0] \Rightarrow y[0] = x[0]$

$y[1] = \frac{1}{2} y[0] + x[1] = \frac{1}{2} x[0] + x[1]$

$y[2] = \frac{1}{2} y[1] + x[2] = \frac{1}{2}(\frac{1}{2}x[0] + x[1]) + x[2]$

$y[3] = \frac{1}{2} y[2] + x[3] = \frac{1}{2}(\frac{1}{2}(\frac{1}{2}x[0] + x[1]) + x[2]) + x[3]$

$\boxed{y[n] = \frac{1}{2^n} x[0] + \frac{1}{2^{n-1}} x[1] + \cdots + \frac{1}{2} x[n-1] + x[n]}$ (*)

if $x[n] = \alpha_1 x_1[n] + \beta x_2[n] \Rightarrow y[n] =$

$y[n] = \frac{1}{2^n}([\alpha x_1[0] + \beta x_2[0]]) + \frac{1}{2^{n-1}}(\alpha x_1[1] + \beta x_2[1]) + \cdots + \frac{1}{2}(\alpha x_1[n-1] + \beta x_2[n-1])$

$+ (\alpha x_1[n] + \beta x_2[n]) \Rightarrow y[n] = \alpha(\frac{1}{2^n} x_1[0] + \frac{1}{2^{n-1}} x[1] + \cdots \frac{1}{2} x[n-1] + x[n])$

$+ \beta(\frac{1}{2^n} x_2[0] + \frac{1}{2^{n-1}} x_2[1] + \cdots \frac{1}{2} x_2[n-1] + x_2[n]) = \alpha y_1[n] + \beta y_2[n]$

linear ✓

$$*: \quad y[n] = \frac{1}{2^n} x[0] + \frac{1}{2^{n-1}} x[1] + \cdots + \frac{1}{2} x[n-1] + x[n]$$

For time invariante we should see:

$$\text{if } x[n] \to x[n-n_0] \overset{?}{\Longrightarrow} y[n] \to y[n-n_0]$$

$$\text{if } x[n] \longrightarrow x[n-n_0] \Rightarrow y_1[n] = \overbrace{\frac{1}{2^n} x[0-n_0] + \frac{1}{2^{n-1}} x[1-n_0] + \cdots}^{0}$$

$$+ \frac{1}{2^{n-n_0}} x[n_0-n_0] + \frac{1}{2^{n-(n_0+1)}} x[(n_0+1)-n_0] + \cdots + \frac{1}{2^1} x[n-1-n_0] + x[n-n_0]$$

$$= \frac{1}{2^{n-n_0}} x[0] + \frac{1}{2^{n-(n_0+1)}} x[1] + \cdots + \frac{1}{2} x\, x[n-1-n_0] + x[n-n_0] \quad (1)$$

on the other hand:

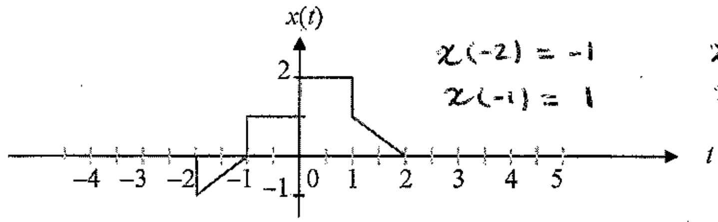
$$(*) \Rightarrow$$

$$y[n-n_0] = \frac{1}{2^{n-n_0}} x[0] + \frac{1}{2^{n-1-n_0}} x[1] + \cdots + \frac{1}{2} x[n-1-n_0] + x[n-n_0] \quad (2)$$

$$\Rightarrow$$

$$(1) = (2) \Rightarrow y[n] \longrightarrow y[n-n_0] \quad \checkmark$$

2. A CT signal $x(t)$ is shown in Figure1



$x(-2) = -1$ $\quad$ $x(0) = 2$

$x(-1) = 1$ $\quad$ $x(1) = 1$

Figure1: Waveform for the CT signal $x(t)$

Sketch and label carefully each of the following signals:

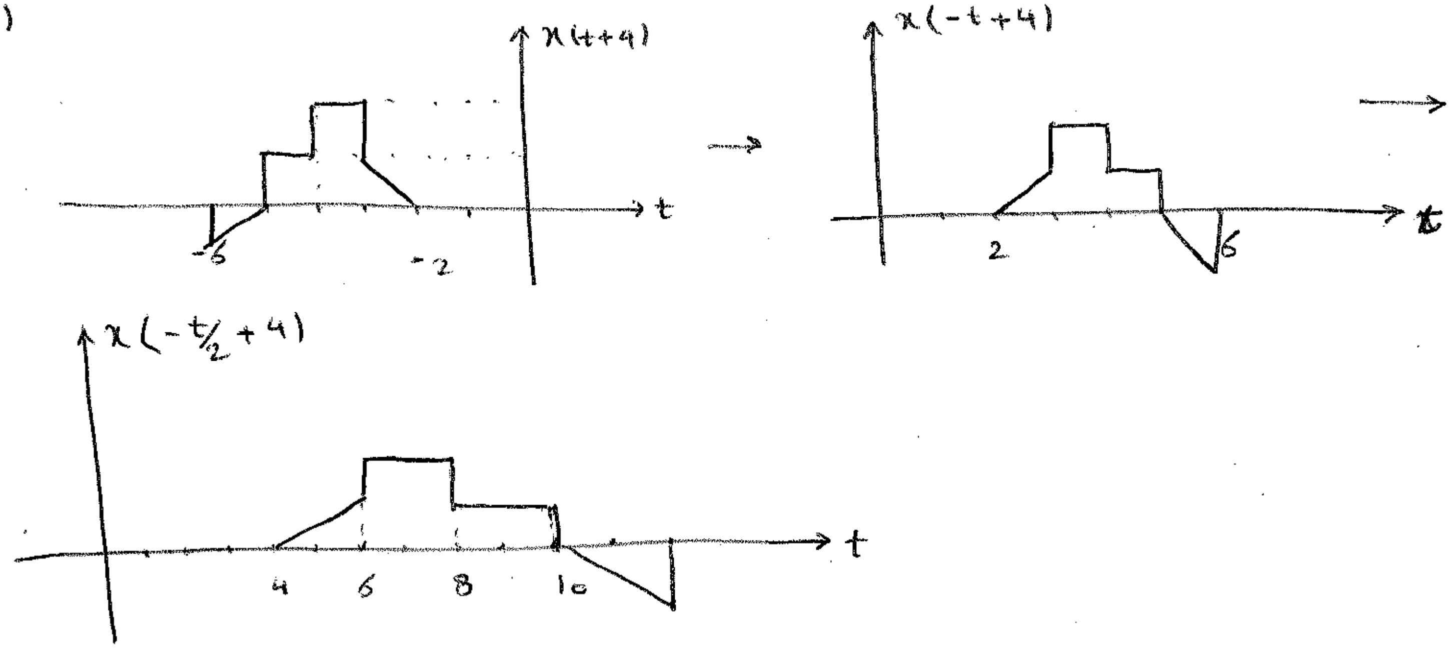
(a) $x(4 - \frac{t}{2})$ [1.5 points]

(b) $x(t).\sum_{k=-\infty}^{\infty} \delta(t - 0.5k)$ [1.5 points]

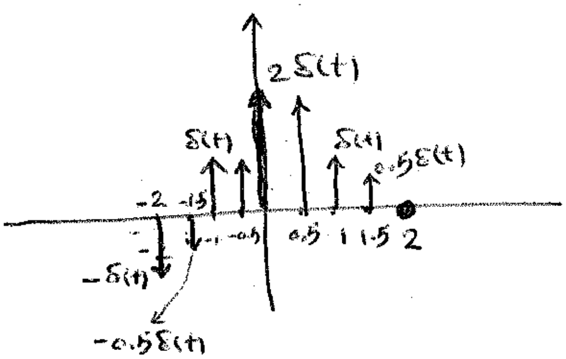
(c) Even component of $x(t)$ [1.5 points].

(d) $x(2t + 1).[u(t + 3) + u(t + 1) - u(t - 1) - u(t - 3)]$ [1.5 points]

a)

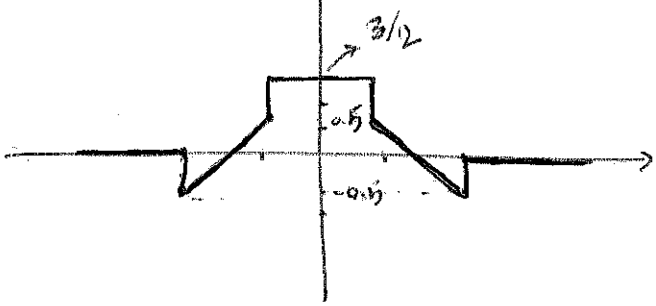


b)

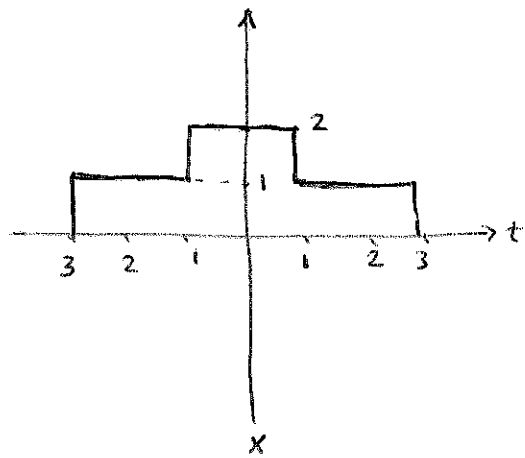


c)

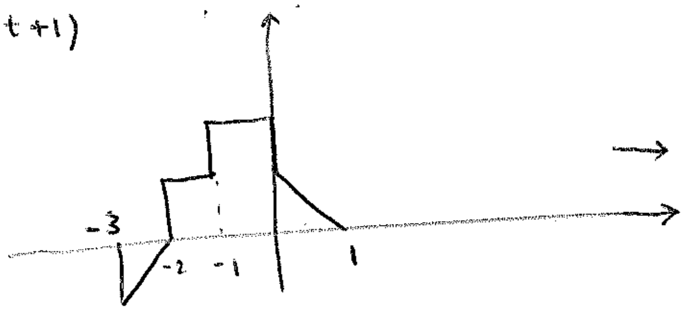
$$Even[x(t)] = \frac{x(t) + x(-t)}{2}$$



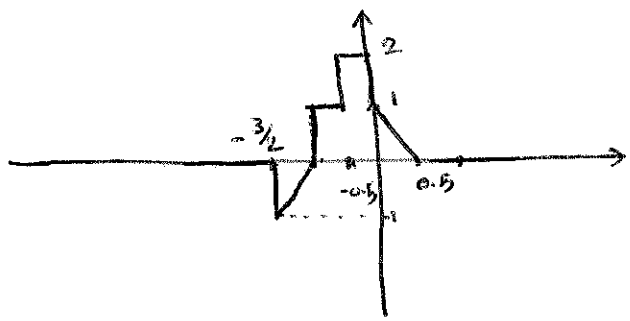
D) $[U(t+3) + U(t+1) - U(t-1) - U(t-3)]$

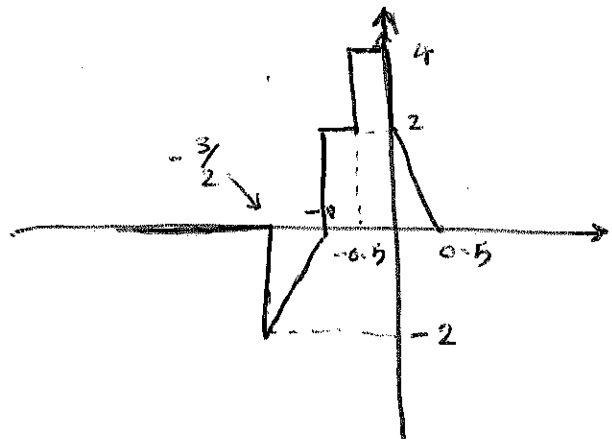


$x(t+1)$



$x(2t+1)$



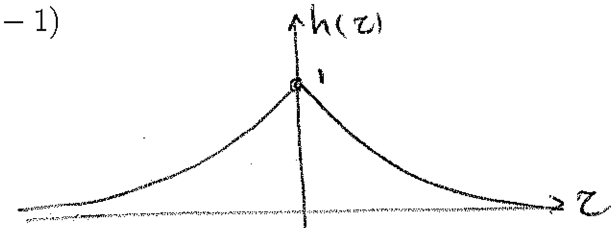


3. Consider a CT LTI system with the impulse response

$$h(t) = e^{-4|t|}$$

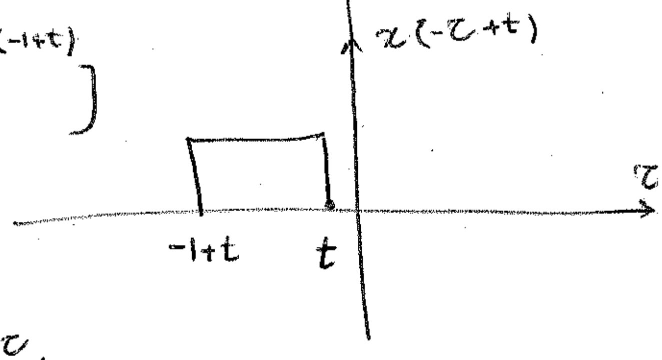
Calculate the output of the LTI system to the following input [6 points].

$$x(t) = u(t) - u(t-1)$$



Por $t<0 \Longrightarrow y(t) = \int_{-1+t}^{t} e^{-4\times(-\tau)} d\tau$

$$= \int_{-1+t}^{t} e^{4\tau} d\tau = \frac{1}{4} e^{4\tau} \Big|_{-1+t}^{t} = \frac{1}{4}\left[e^{4t} - e^{4(-1+t)}\right]$$



Por $0<t<1 \Rightarrow y(t) = \int_{-1+t}^{0} e^{4\tau} d\tau + \int_{0}^{t} e^{-4\tau} d\tau$

$$= \frac{1}{4}\left[e^{4\tau}\right]_{0}^{-1+t} + \left(-\frac{1}{4}\right)\left[e^{-4\tau}\right]_{0}^{t} = \frac{1}{4}\left[e^{4(t-1)} - e^{4t}\right]$$

Por $t>1 \Rightarrow y(t) = \int_{-1+t}^{t} e^{-4\tau} d\tau = \left(-\frac{1}{4}\right)\left[e^{-4\tau}\right]_{-1+t}^{t} = -\frac{1}{4}\left[e^{-4t} - e^{-4(-1+t)}\right.$

4. Consider a DT LTI system with the impulse response

$$h[n] = \begin{cases} 2 & n \geq 0 \\ 0 & otherwise \end{cases}$$

The signal at the input is

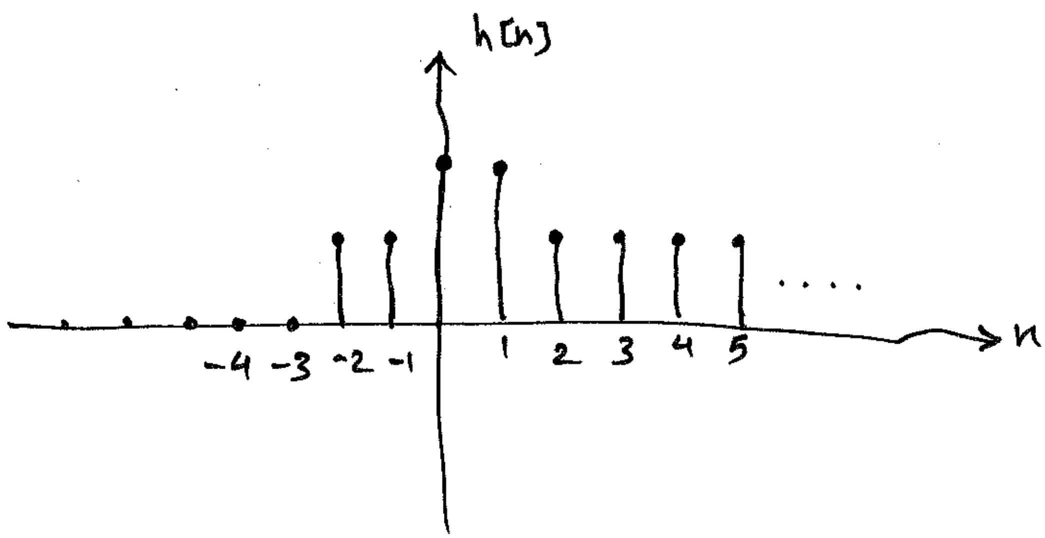
$$x[n] = \delta[n+2] + \delta[n] - \delta[n-2]$$

where $\delta[n]$ is the unit impulse function.

Derive the expression of the signal at the output of the system, $y[n]$. Sketch carefully $y[n]$ [6 points].

$y[n] = x[n] * h[n] = (\delta[n+2] + \delta[n] - \delta[n-2]) * h[n]$

$= h[n+2] + h[n] - h[n-2] \Longrightarrow$



5. Consider the following system in Figure2:

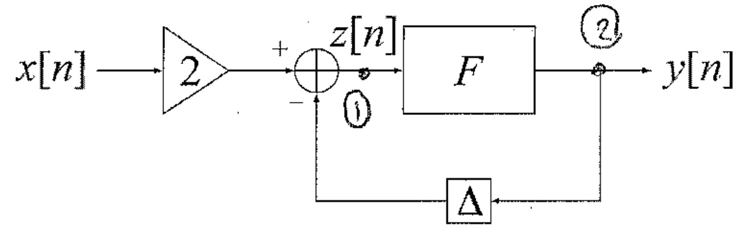


Figure2: System Block Diagram

Here, the system F is defined by the input-output relationship

$$F\{z[n]\} = y[n] = z[n] - z[n-1]$$

and Δ is the unit delay

$$\Delta\{w[n]\} = w[n-2]$$

Write down the linear difference equation describing this system [6 points].

1 $x[n] \times 2 - \Delta\{y[n]\} = z[n] \Rightarrow z[n] = 2x[n] - y[n-2]$ *

2 $y[n] = F\{z[n]\} \overset{*}{\Rightarrow} y[n] = 2x[n] - y[n-2] - [2x[n-1] - y[n-1-2]] \Rightarrow y[n] = 2x[n] - y[n-2] - 2x[n-1] + y[n-3]$

$\Rightarrow \boxed{y[n] = y[n-3] - y[n-2] + 2x[n] - 2x[n-1]}$

$\Delta = 2$

or $y[n] = y[n-2] - y[n-1] + 2x[n] - 2x[n-1]$

for $\Delta = 1$