

## MSE 222 DYNAMICS Midterm 1 Exam

SIMON FRASER UNIVERSITY  
MECHATRONIC SYSTEMS ENGINEERING

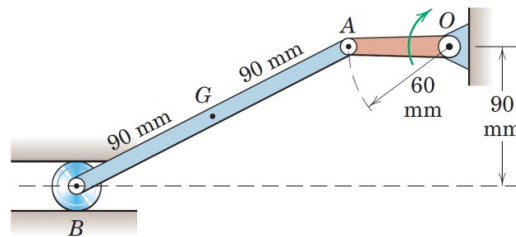
Midterm 1 Examination – February 18, 2016

Instructor: Kambiz Hajikolaie

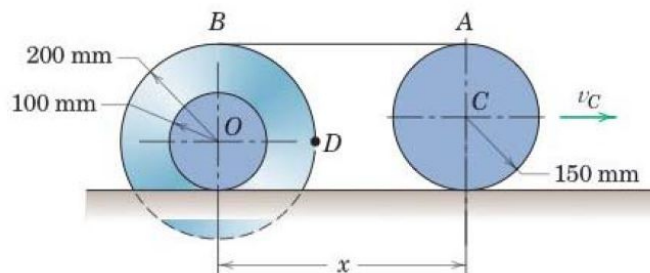
Time: 90 minutes

Non-programmable calculators may be used  
No smartphones or other electronic devices may be used  
Answer all the questions in the booklet (not the question sheet)

**Problem1:** In the mechanism below, crank OA is rotating with constant angular velocity of  $\omega$ . For the instant represented, when crank OA passes the horizontal position, determine the velocity of the center G of link AB by Instantaneous Center of Zero Velocity (IC) method, as a function of  $\omega$ .

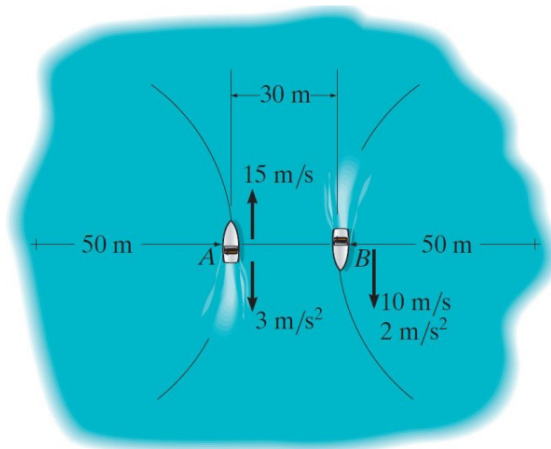


**Problem2:** Determine the magnitude of the acceleration of point D in the position shown if the center C of the smaller wheel has acceleration to the right of  $0.8 \frac{m}{s^2}$  and has reached a velocity of  $0.4 \frac{m}{s}$  at this instant. The cord which connects the two wheels is securely wrapped around the respective peripheries and does not slip.

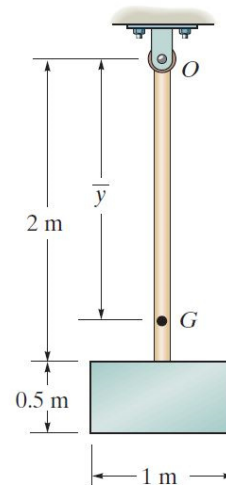


**Problem3:** At the instant shown, boat A travels with a speed of  $15 \text{ m/s}$ , which is decreasing at  $3 \text{ m/s}^2$ , while boat B travels with a speed of  $10 \text{ m/s}$ , which is increasing at  $2 \text{ m/s}^2$ .

- Determine the velocity and acceleration of boat A with respect to boat B at this instant.
- Determine the velocity and acceleration of boat B with respect to boat A at this instant.



**Problem4:** The pendulum consists of the 3-kg slender rod and the 5-kg thin plate. Calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through G (center of mass).



**Kinematics Formulas:**

Rigid-body analysis:

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

Relative motion:

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\Omega} \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{B/A} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{B/A}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

**Moment of Inertia Formulas:**

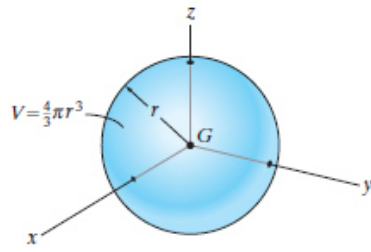
General Formula

$$I = \int_m r^2 dm$$

Parallel-axis theorem

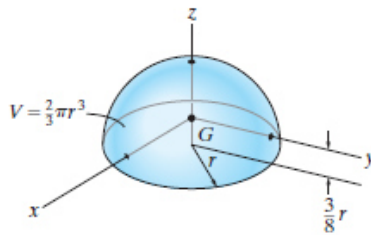
$$I = I_G + md^2$$

## Center of Gravity and Mass Moment of Inertia of Homogeneous Solids



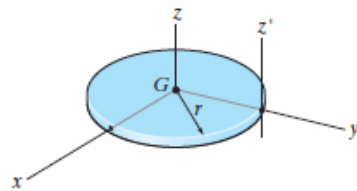
Sphere

$$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} mr^2$$



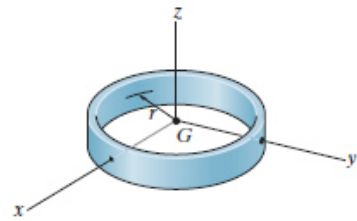
Hemisphere

$$I_{xx} = I_{yy} = 0.259 mr^2 \quad I_{zz} = \frac{2}{5} mr^2$$



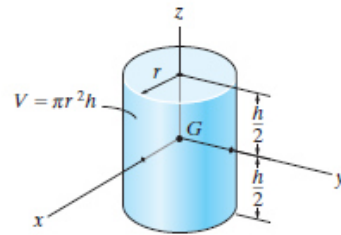
Thin Circular disk

$$I_{xx} = I_{yy} = \frac{1}{4} mr^2 \quad I_{zz} = \frac{1}{2} mr^2 \quad I_{z'z'} = \frac{3}{2} mr^2$$



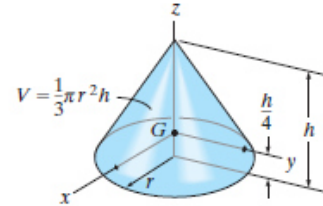
Thin ring

$$I_{xx} = I_{yy} = \frac{1}{2} mr^2 \quad I_{zz} = mr^2$$



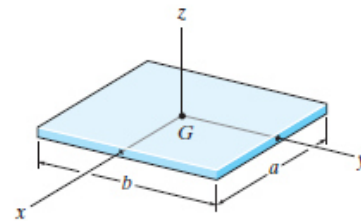
Cylinder

$$I_{xx} = I_{yy} = \frac{1}{12} m(3r^2 + h^2) \quad I_{zz} = \frac{1}{2} mr^2$$



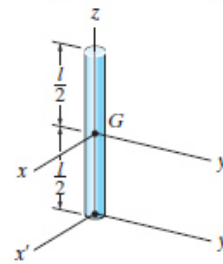
Cone

$$I_{xx} = I_{yy} = \frac{3}{80} m(4r^2 + h^2) \quad I_{zz} = \frac{3}{10} mr^2$$



Thin plate

$$I_{xx} = \frac{1}{12} mb^2 \quad I_{yy} = \frac{1}{12} ma^2 \quad I_{zz} = \frac{1}{12} m(a^2 + b^2)$$



Slender Rod

$$I_{xx} = I_{yy} = \frac{1}{12} ml^2 \quad I_{z'z'} = I_{y'y'} = \frac{1}{3} ml^2 \quad I_{z'z'} = 0$$