MSE 222 DYNAMICS Final Exam

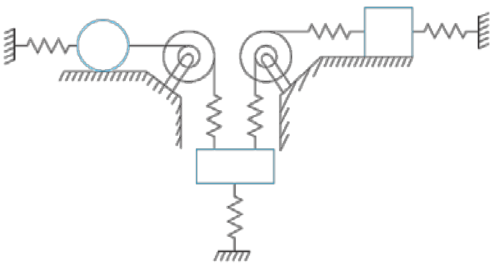
SIMON FRASER UNIVERSITY
MECHATRONIC SYSTEMS ENGINEERING
Final Examination – April 17, 2016

Instructor: Kambiz Hajikolaei   Time: 150 minutes

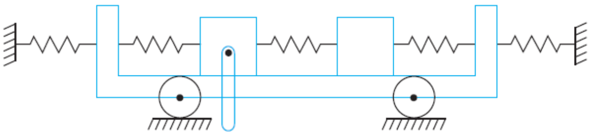
Non-programmable calculators may be used
No smartphones or other electronic devices may be used
Answer all the questions in the booklet (not the question sheet)

Section I: short answer questions (10 marks)

Q1. Determine the number of degrees of freedom in the following dynamic systems:



(a)



(b)

Q2. Define the following terms in a single-degree-of-freedom vibrational system:

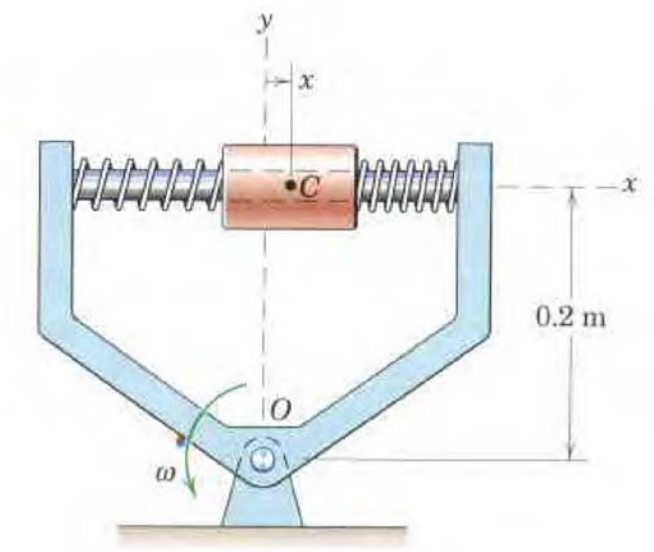
a) Period of oscillation
b) Amplitude
c) Phase angle
d) Critically damped motion

Q3. True-False questions:

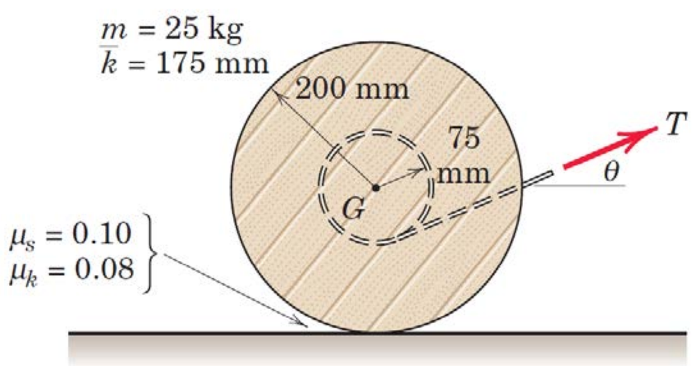
a) Free vibration takes place at the system's natural frequency, irrespective of the initial conditions.
b) Natural frequencies depend only on the stiffness properties of the system.
c) Natural frequency decreases with an increase in the stiffness or a decrease in the mass.
d) When an underdamped system is disturbed from rest, the motion is non-periodic.

Section II: Problems

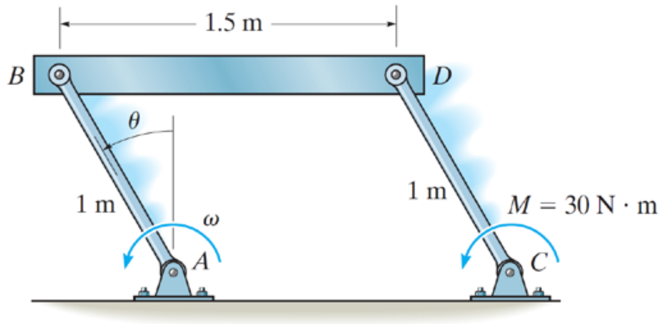
Problem1: The spring-mounted collar oscillates on the shaft according to $x = 0.04\sin(\pi t)$, where x is in meters and t is in seconds. Simultaneously the frame rotates about the bearing at O with an angular velocity $\omega = 2\sin\left(\frac{\pi t}{2}\right)$ rad/s. Determine the acceleration of the center C of the collar (a) when t=3s and (b)when t=0.5s. (25 marks)



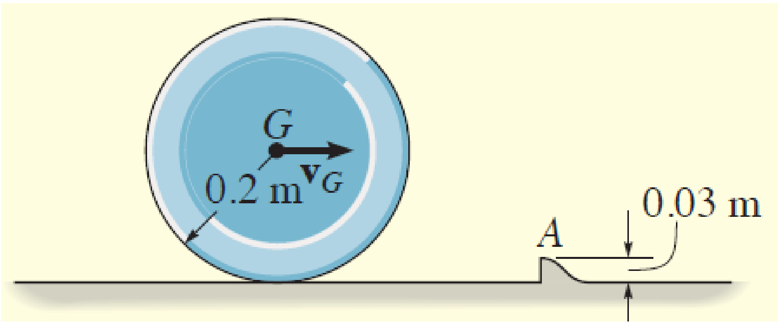
Problem2: The circular disk of 200-mm radius has a mass of 25 kg with centroidal radius of gyration $k = 175\ mm$ and has a concentric circular groove of $75\ mm$ radius cut into it. A steady force T is applied at an angle θ to a cord wrapped around the groove as shown. If $T = 30\ N$, $\theta = 0$, $\mu_s = 0.1$ and $\mu_k = 0.08$, determine the angular acceleration α of the disk, the acceleration a of its mass center G, and the friction force F which the surface exerts on the disk. (20 marks)



Problem3: The linkage consists of two 6-kg rods AB and CD and a 20-kg bar BD. When $\theta = 0°$, rod AB is rotating with an angular velocity $\omega = 2\ rad/s$. If rod CD is subjected to a couple moment of M = 30 N-m, determine ω_{AB} at the instant $\theta = 90°$. (20 marks)



Problem4: The 10-kg wheel shown in the figure has a moment of inertia $I_G = 0.156\ kg.m^2$. Assuming that the wheel does not slip or rebound, determine the minimum velocity v_G it must have to just roll over the obstruction at A. (25 marks)



Kinematics Formulas:
Rigid-body analysis:
$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$ $\quad\quad$ $\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$
Relative motion:
$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\Omega} \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$
$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{B/A} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{B/A}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$

Moment of Inertia Formulas:	
General Formula $I = \int_m r^2\, dm$	Parallel-axis theorem $I = I_G + md^2$

Kinetics Formulas:	
$\Sigma F_x = m(a_G)_x \quad \Sigma F_y = m(a_G)_y \quad \Sigma M_G = 0$ Rectilinear translation	$\Sigma F_n = m(a_G)_n \quad \Sigma F_t = m(a_G)_t \quad \Sigma M_G = 0$ Curvilinear translation
$\Sigma F_n = m(a_G)_n = m\omega^2 r_G$ $\Sigma F_t = m(a_G)_t = m\alpha r_G$ $\Sigma M_G = I_G\alpha$ or $\Sigma M_O = I_O\alpha$ Rotation about a fixed axis	$\Sigma F_x = m(a_G)_x$ $\Sigma F_y = m(a_G)_y$ $\Sigma M_G = I_G\alpha$ or $\Sigma M_P = \Sigma(\mathcal{M}_k)_P$ General plane motion

Work and Energy Formulas:	
Kinetic Energy (Translation): $T = \frac{1}{2} m v_G^2$	
Kinetic Energy(General Plane Motion): $T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2 \;\; or \;\; \frac{1}{2} I_{IC} \omega^2$	Kinetic Energy (Rotation about a fixed axis): $T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2 \;\; or \;\; \frac{1}{2} I_O \omega^2$
Principle of work and energy: $T_1 + \Sigma U_{1-2} = T_2$	Conservation of energy: $T_1 + V_1 = T_2 + V_2$ where $V = V_g + V_e$

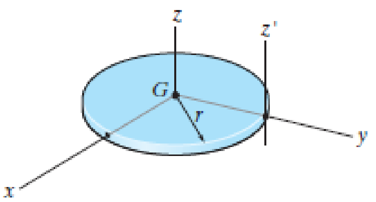
Impulse and Momentum Formulas:		
Linear and Angular Momentum:		
Translation	Rotation about a fixed axis	General plane motion
$L = m v_G$	$L = m v_G$	$L = m v_G$
$H_G = 0$	$H_G = I_G \omega$	$H_G = I_G \omega$
$H_A = (m v_G) d$	$H_O = I_O \omega$	$H_A = I_G \omega + (m v_G) d$

Principle of Impulse and Momentum	Conservation of Momentum
$m(v_{Gx})_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m(v_{Gx})_2$	$\left(\Sigma \begin{matrix} \text{syst. linear} \\ \text{momentum} \end{matrix} \right)_1 = \left(\Sigma \begin{matrix} \text{syst. linear} \\ \text{momentum} \end{matrix} \right)_2$
$m(v_{Gy})_1 + \Sigma \int_{t_1}^{t_2} F_y \, dt = m(v_{Gy})_2$	$\left(\Sigma \begin{matrix} \text{syst. angular} \\ \text{momentum} \end{matrix} \right)_{O1} = \left(\Sigma \begin{matrix} \text{syst. angular} \\ \text{momentum} \end{matrix} \right)_{O2}$
$I_G \omega_1 + \Sigma \int_{t_1}^{t_2} M_G \, dt = I_G \omega_2$	

Coefficient of Restitution:

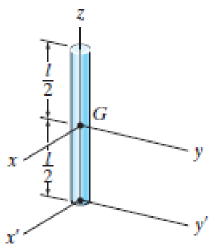
$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

Center of gravity and mass moment of inertia of homogeneous solids



Thin Circular disk

$I_{xx} = I_{yy} = \frac{1}{4} m r^2 \quad I_{zz} = \frac{1}{2} m r^2 \quad I_{z'z'} = \frac{3}{2} m r^2$

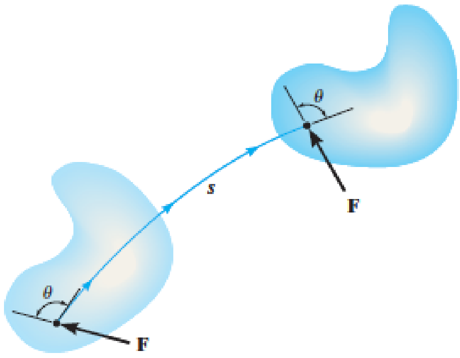


Slender Rod

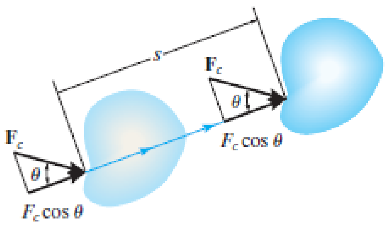
$I_{xx} = I_{yy} = \frac{1}{12} m l^2 \quad I_{x'x'} = I_{y'y'} = \frac{1}{3} m l^2 \quad I_{z'z'} = 0$

Work of a Force and a Couple Moment

A force does work when it undergoes a displacement ds in the direction of the force. In particular, the frictional and normal forces that act on a cylinder or any circular body that rolls *without slipping* will do no work, since the normal force does not undergo a displacement and the frictional force acts on successive points on the surface of the body.

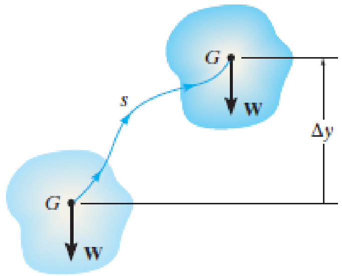


$$U_F = \int F \cos\theta \, ds$$



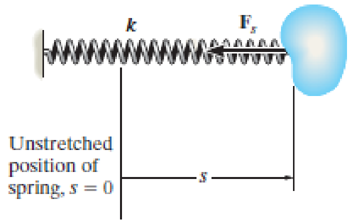
$$U_{F_c} = (F_c \cos\theta)s$$

Constant Force



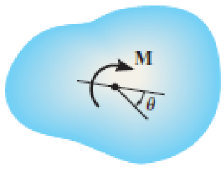
$$U_W = -W\Delta y$$

Weight



$$U = -\frac{1}{2}k s^2$$

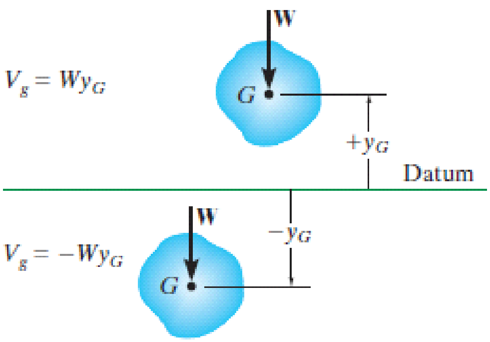
Spring



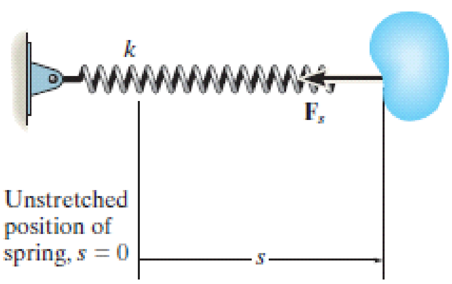
$$U_M = \int_{\theta_1}^{\theta_2} M \, d\theta$$

$$U_M = M(\theta_2 - \theta_1)$$

Constant Magnitude



Gravitational potential energy



$$V_e = \frac{1}{2}ks^2$$

Elastic potential energy