MSE 210 – Engineering Measurement and Data Analysis

Midterm (Spring 2018)

Name: Solutions

Student Number:____________________

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Duration: 1:50min

There are 4 questions and 6 pages on this examination (the last 2 pages are appendices). The midterm is worth 40 pts. Please ensure that you have all pages before starting this examination.

Q1) A specimen is subjected to stress cycling at a maximum stress amplitude. Let the random variable be the number of cycles to failure, which follows a Weibull distribution with parameters $\beta = 0.25$ and $\delta = 1{,}000$ cycles.

a) What is the probability that the specimen does not exceed 15,000 cycles? (2 pts)

$$P(X < 15{,}000) = F(15{,}000) = 1 - e^{-\left(\frac{x}{\delta}\right)^{\beta}}$$

$$= 1 - e^{-\left(\frac{15{,}000}{1{,}000}\right)^{0.25}} = 1 - e^{-1.968} = 0.8603$$

86% of the specimens will not exceed 15,000 cycles

b) Determine the probability that the specimen exceeds twice as many cycles as its mean. (4 pts)

Find the mean of the distribution. For a Weibull distribution

$$\mu = \delta\,\Gamma\left(1 + \frac{1}{\beta}\right) = 1{,}000\,\Gamma\left(1 + \frac{1}{0.25}\right) = 1{,}000\,\Gamma(5)$$

$$= 1{,}000\,(5-1)! = 24{,}000 \text{ cycles}$$

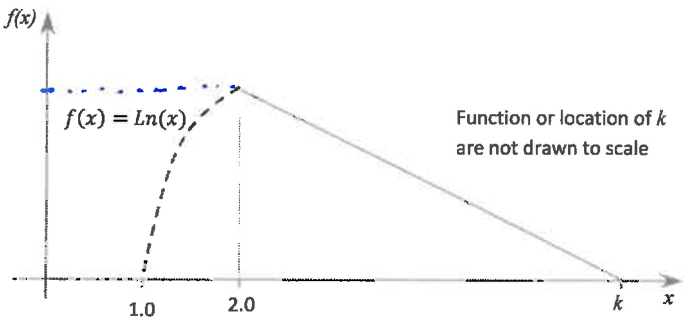
$$P(X > 48{,}000) = 1 - F(48{,}000) = 1 - \left(1 - e^{-\left(\frac{x}{\delta}\right)^{\beta}}\right) = e^{-\left(\frac{x}{\delta}\right)^{\beta}}$$

$$= e^{-\left(\frac{48{,}000}{1000}\right)^{0.25}} = 0.0719$$

7.2% of the specimens will exceed 48,000 cycles

Q2) Shown below is a partially complete probability density function.

$h = Ln(2)$
$= 0.6931$



$$\int Ln(x) = xLn(x) - x + C$$

a) Determine the slope of the line. (7 pts)

Area under $f(x) = Ln(x)$

$$\text{area} = \int_1^2 Ln(x)\,dx = x\,Ln(x) - x \Big|_1^2$$

$$= 2Ln(2) - 2 - (Ln(1) - 1) = 0.3862$$

Area under triangle

$$\text{area} = 1 - 0.3862 = \frac{b \times h}{2} \Rightarrow b = \frac{2(0.6138)}{0.6931} = 1.7708$$

$k = 2 + 1.7708 = 3.7708$

$$m = \frac{y - y_0}{x - x_0} = \frac{0 - 0.6931}{3.7708 - 2} = -0.391436$$

$m = -0.391436$

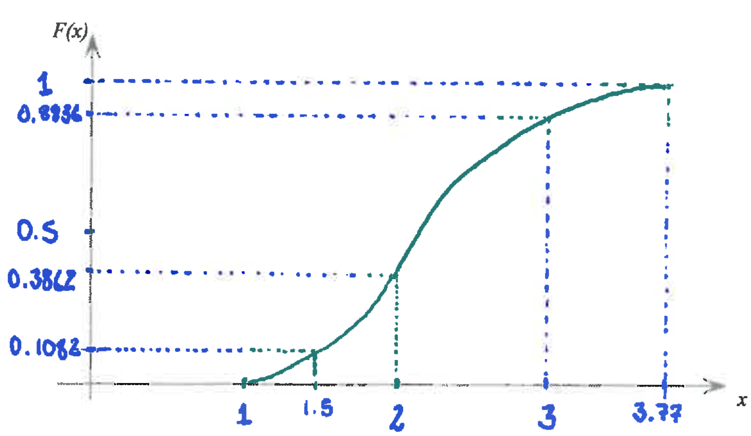
b) Define the probability density function and draw the cumulative density function (5 pts)

$$f(x) = \begin{cases} Ln(x) & for\ 1 < x < 2 \\ -0.3914x + 1.476 & \text{for } 2 < X < 3.7708 \\ 0 & \text{otherwise} \end{cases}$$

$$y = m(x - x_0) + y_0 = -0.391436(x-2) + 0.6931$$
$$= -0.391436x + 1.47602$$

$$P(1 < x < 1.5) = 1.5\,Ln(1.5) - 1.5 - (Ln(1) - 1) = 0.1082$$

$$P(1 < x < 3) = 0.3862 + \int_2^3 -0.3914x + 1.476\,dx$$
$$= 0.3862 + \frac{(-0.3914)x^2}{2} + 1.476x \Big|_2^3$$
$$= 0.8836$$



Q3) A company produces shafts of different lengths. Their quality control states that a 75cm long shaft must be rejected if it has two or more imperfections. The company has determined that the number of imperfections on the shaft follows a Poisson distribution with 0.6 imperfections in every meter of shaft extrusion.

a) What is the probability that there are two or more imperfections in a 75cm shaft? (4 pts)

$$E(x) = \lambda = 0.6 \text{ imperf}/m \times 0.75 \text{ m} = 0.45 \text{ imperfections}$$

$$P(x \geq 2) = 1 - P(x=0) - P(x=1) \qquad p(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= 1 - \frac{e^{-0.45}\,0.45^{0}}{0!} - \frac{e^{-0.45}\,0.45^{1}}{1!} = 1 - 0.6376 - 0.2869 = 0.07544$$

The probability that a shaft gets rejected (two or more imperfections) is 7.5%

b) What is the probability that three shafts are rejected in a batch of 20? If unable to find the probability of rejection, use p = 0.1 but one point will be deducted. (3 pts)

This is a binomial distribution (failure vs. success), with $p = 0.07544$, $n=20$ and $x=3$.

$$\binom{n}{x} = \frac{n!}{x!\,(n-x)!} = \frac{20!}{3!\,(17!)} = \frac{20\cdot 19\cdot 18\cdot \cancel{17!}}{3!\,(\cancel{17!})} = 1{,}140$$

$$P(x=3) = \binom{n}{x} p^{x}(1-p)^{n-x} = 1{,}140\,(0.07544)^{3}(1-0.07544)^{20-3} = 0.129$$

The probability that 3 shafts are rejected in a batch of 20 is 12.9%

c) What is the probability that more than 10 shafts are rejected in a batch of 100? Use the normal approximation approach. One point will be deducted if you use p = 0.1. (3 pts)

continuity correction

$$P(x > 10) = 1 - P\left(z < \frac{x + 0.5 - np}{\sqrt{np(1-p)}}\right) = 1 - P\left(z < \frac{10.5 - 7.544}{\sqrt{7.544\,(1-0.07544)}}\right)$$

$$= 1 - P(z < 1.1193) = 1 - 0.8686 = 0.1314$$

The probability that more than 10 shaft are rejected in a batch of 100 is 13.14%

$q_2 = 21.3$

$r_1 = 0.25(n-1) = 0.25(10) = 2.5$

$q_1 = 19.5 + (20.9 - 19.5)(0.5) = 20.2$

$r_3 = 0.75(10) = 7.5$

$q_3 = 23.2 + (25.2 - 23.2)(0.5) = 24.2$

$IQR \times 1.5 = (24.2 - 20.2) \times 1.5 = 6$

Whiskers

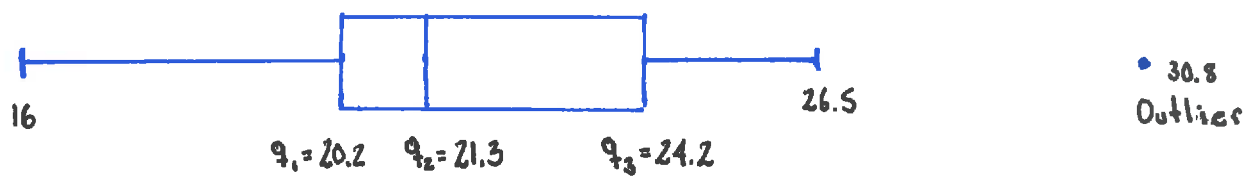
Lower : $20.6 - 6 = 14.6$

Upper = $24.2 + 6 = 30.2$

Q4) A company is testing the combined city/hwy fuel efficiency of a new SUV. Eleven random samples at different driving conditions, from heavy city traffic to highway, were recorded:

16	17	19.5	20.9	21	21.3	22.8	23.2	25.2	26.5	30.8
0	1	2	3	4	5	6	7	8	9	10

a) Determine the boxplot using the inclusive method. Use a scale of 1 cm = 1 mpg. (5 pts)



b) Assume the data follows a normal distribution. Test the hypothesis that the new SUV has a better fuel efficiency than the previous model at 19.9 mpg. The sample mean and sample standard deviation are 22.2 and 4.237 mpg, respectively. Let α = 0.05. (5 pts)

1. Parameter of interest: mean of the population
2. Establish Hypothesis: H0: $\mu \leq 19.9$ and H1: $\mu > 19.9$ (Claim) Upper tail test
3. Test Statistic: $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$
4. Type I error: $\alpha = 0.05$
5. Decision Rule: Reject H_0 if $t_0 > t_{\alpha, n-1}$, where $t_{0.05, 10} = 1.812$
6. Calculate Test Statistic

$$t_0 = \frac{22.2 - 19.9}{4.237/\sqrt{11}} = 1.800$$

7. Draw Conclusions:

Since $t_0 < t_{0.05, 10}$, there is no strong evidence that H_0 can be rejected.

a) Find the 95% confidence interval on the population mean. (2 pts)

$$\bar{x} - t_{0.05, 10}\, s/\sqrt{n} \leq \mu$$

$$22.2 - 1.812\left(4.237/\sqrt{11}\right) \leq \mu$$

$$19.88 \leq \mu$$

Also the CI at 95% shows that H_0 cannot be rejected.

Appendix A. Table I: Cumulative Standard Normal Distribution

z	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	-0.00	z
−3.9	0.000033	0.000034	0.000036	0.000037	0.000039	0.000041	0.000042	0.000044	0.000046	0.000048	−3.9
−3.8	0.000050	0.000052	0.000054	0.000057	0.000059	0.000062	0.000064	0.000067	0.000069	0.000072	−3.8
−3.7	0.000075	0.000078	0.000082	0.000085	0.000088	0.000092	0.000096	0.000100	0.000104	0.000108	−3.7
−3.6	0.000112	0.000117	0.000121	0.000126	0.000131	0.000136	0.000142	0.000147	0.000153	0.000159	−3.6
−3.5	0.000165	0.000172	0.000179	0.000185	0.000193	0.000200	0.000208	0.000216	0.000224	0.000233	−3.5
−3.4	0.000242	0.000251	0.000260	0.000270	0.000280	0.000291	0.000302	0.000313	0.000325	0.000337	−3.4
−3.3	0.000350	0.000362	0.000376	0.000390	0.000404	0.000419	0.000434	0.000450	0.000467	0.000483	−3.3
−3.2	0.000501	0.000519	0.000538	0.000557	0.000577	0.000598	0.000619	0.000641	0.000664	0.000687	−3.2
−3.1	0.000711	0.000736	0.000762	0.000789	0.000816	0.000845	0.000874	0.000904	0.000935	0.000968	−3.1
−3.0	0.001001	0.001035	0.001070	0.001107	0.001144	0.001183	0.001223	0.001264	0.001306	0.001350	−3.0
−2.9	0.001395	0.001441	0.001489	0.001538	0.001589	0.001641	0.001695	0.001750	0.001807	0.001866	−2.9
−2.8	0.001926	0.001988	0.002052	0.002118	0.002186	0.002256	0.002327	0.002401	0.002477	0.002555	−2.8
−2.7	0.002635	0.002718	0.002803	0.002890	0.002980	0.003072	0.003167	0.003264	0.003364	0.003467	−2.7
−2.6	0.003573	0.003681	0.003793	0.003907	0.004025	0.004145	0.004269	0.004396	0.004527	0.004661	−2.6
−2.5	0.004799	0.004940	0.005085	0.005234	0.005386	0.005543	0.005703	0.005868	0.006037	0.006210	−2.5
−2.4	0.006387	0.006569	0.006756	0.006947	0.007143	0.007344	0.007549	0.007760	0.007976	0.008198	−2.4
−2.3	0.008424	0.008656	0.008894	0.009137	0.009387	0.009642	0.009903	0.010170	0.010444	0.010724	−2.3
−2.2	0.011011	0.011304	0.011604	0.011911	0.012224	0.012545	0.012874	0.013209	0.013553	0.013903	−2.2
−2.1	0.014262	0.014629	0.015003	0.015386	0.015778	0.016177	0.016586	0.017003	0.017429	0.017864	−2.1
−2.0	0.018309	0.018763	0.019226	0.019699	0.020182	0.020675	0.021178	0.021692	0.022216	0.022750	−2.0
−1.9	0.023295	0.023852	0.024419	0.024998	0.025588	0.026190	0.026803	0.027429	0.028067	0.028717	−1.9
−1.8	0.029379	0.030054	0.030742	0.031443	0.032157	0.032884	0.033625	0.034379	0.035148	0.035930	−1.8
−1.7	0.036727	0.037538	0.038364	0.039204	0.040059	0.040929	0.041815	0.042716	0.043633	0.044565	−1.7
−1.6	0.045514	0.046479	0.047460	0.048457	0.049471	0.050503	0.051551	0.052616	0.053699	0.054799	−1.6
−1.5	0.055917	0.057053	0.058208	0.059380	0.060571	0.061780	0.063008	0.064256	0.065522	0.066807	−1.5
−1.4	0.068112	0.069437	0.070781	0.072145	0.073529	0.074934	0.076359	0.077804	0.079270	0.080757	−1.4
−1.3	0.082264	0.083793	0.085343	0.086915	0.088508	0.090123	0.091759	0.093418	0.095098	0.096801	−1.3
−1.2	0.098525	0.100273	0.102042	0.103835	0.105650	0.107488	0.109349	0.111233	0.113140	0.115070	−1.2
−1.1	0.117023	0.119000	0.121001	0.123024	0.125072	0.127143	0.129238	0.131357	0.133500	0.135666	−1.1
−1.0	0.137857	0.140071	0.142310	0.144572	0.146859	0.149170	0.151505	0.153864	0.156248	0.158655	−1.0
−0.9	0.161087	0.163543	0.166023	0.168528	0.171056	0.173609	0.176185	0.178786	0.181411	0.184060	−0.9
−0.8	0.186733	0.189430	0.192150	0.194894	0.197662	0.200454	0.203269	0.206108	0.208970	0.211855	−0.8
−0.7	0.214764	0.217695	0.220650	0.223627	0.226627	0.229650	0.232695	0.235762	0.238852	0.241964	−0.7
−0.6	0.245097	0.248252	0.251429	0.254627	0.257846	0.261086	0.264347	0.267629	0.270931	0.274253	−0.6
−0.5	0.277595	0.280957	0.284339	0.287740	0.291160	0.294599	0.298056	0.301532	0.305026	0.308538	−0.5
−0.4	0.312067	0.315614	0.319178	0.322758	0.326355	0.329969	0.333598	0.337243	0.340903	0.344578	−0.4
−0.3	0.348268	0.351973	0.355691	0.359424	0.363169	0.366928	0.370700	0.374484	0.378281	0.382089	−0.3
−0.2	0.385908	0.389739	0.393580	0.397432	0.401294	0.405165	0.409046	0.412936	0.416834	0.420740	−0.2
−0.1	0.424655	0.428576	0.432505	0.436441	0.440382	0.444330	0.448283	0.452242	0.456205	0.460172	−0.1
0.0	0.464144	0.468119	0.472097	0.476078	0.480061	0.484047	0.488033	0.492022	0.496011	0.500000	0.0

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.523922	0.527903	0.531881	0.535856	0.0
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345	0.1
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092	0.2
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732	0.3
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933	0.4
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405	0.5
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903	0.6
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236	0.7
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267	0.8
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913	0.9
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143	1.0
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977	1.1
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475	1.2
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736	1.3
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888	1.4
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083	1.5
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486	1.6
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273	1.7
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621	1.8
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705	1.9
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691	2.0
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738	2.1
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989	2.2
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576	2.3
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613	2.4
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201	2.5
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427	2.6
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365	2.7
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074	2.8
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605	2.9
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999	3.0
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289	3.1
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499	3.2
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650	3.3
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758	3.4
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835	3.5
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888	3.6
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925	3.7
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950	3.8
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967	3.9

Table II. Normal Error Integral

Table A. The percentage probability, $Prob(\text{within } t\sigma) = \int_{X-t\sigma}^{X+t\sigma} G_{X,\sigma}(x)\,dx$, as a function of t.

t	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00	0.80	1.60	2.39	3.19	3.99	4.78	5.58	6.38	7.17
0.1	7.97	8.76	9.55	10.34	11.13	11.92	12.71	13.50	14.28	15.07
0.2	15.85	16.63	17.41	18.19	18.97	19.74	20.51	21.28	22.05	22.82
0.3	23.58	24.34	25.10	25.86	26.61	27.37	28.12	28.86	29.61	30.35
0.4	31.08	31.82	32.55	33.28	34.01	34.73	35.45	36.16	36.88	37.59
0.5	38.29	38.99	39.69	40.39	41.08	41.77	42.45	43.13	43.81	44.48
0.6	45.15	45.81	46.47	47.13	47.78	48.43	49.07	49.71	50.35	50.98
0.7	51.61	52.23	52.85	53.46	54.07	54.67	55.27	55.87	56.46	57.05
0.8	57.63	58.21	58.78	59.35	59.91	60.47	61.02	61.57	62.11	62.65
0.9	63.19	63.72	64.24	64.76	65.28	65.79	66.29	66.80	67.29	67.78
1.0	68.27	68.75	69.23	69.70	70.17	70.63	71.09	71.54	71.99	72.43
1.1	72.87	73.30	73.73	74.15	74.57	74.99	75.40	75.80	76.20	76.60
1.2	76.99	77.37	77.75	78.13	78.50	78.87	79.23	79.59	79.95	80.29
1.3	80.64	80.98	81.32	81.65	81.98	82.30	82.62	82.93	83.24	83.55
1.4	83.85	84.15	84.44	84.73	85.01	85.29	85.57	85.84	86.11	86.38
1.5	86.64	86.90	87.15	87.40	87.64	87.89	88.12	88.36	88.59	88.82
1.6	89.04	89.26	89.48	89.69	89.90	90.11	90.31	90.51	90.70	90.90
1.7	91.09	91.27	91.46	91.64	91.81	91.99	92.16	92.33	92.49	92.65
1.8	92.81	92.97	93.12	93.28	93.42	93.57	93.71	93.85	93.99	94.12
1.9	94.26	94.39	94.51	94.64	94.76	94.88	95.00	95.12	95.23	95.34
2.0	95.45	95.56	95.66	95.76	95.86	95.96	96.06	96.15	96.25	96.34
2.1	96.43	96.51	96.60	96.68	96.76	96.84	96.92	97.00	97.07	97.15
2.2	97.22	97.29	97.36	97.43	97.49	97.56	97.62	97.68	97.74	97.80
2.3	97.86	97.91	97.97	98.02	98.07	98.12	98.17	98.22	98.27	98.32
2.4	98.36	98.40	98.45	98.49	98.53	98.57	98.61	98.65	98.69	98.72
2.5	98.76	98.79	98.83	98.86	98.89	98.92	98.95	98.98	99.01	99.04
2.6	99.07	99.09	99.12	99.15	99.17	99.20	99.22	99.24	99.26	99.29
2.7	99.31	99.33	99.35	99.37	99.39	99.40	99.42	99.44	99.46	99.47
2.8	99.49	99.50	99.52	99.53	99.55	99.56	99.58	99.59	99.60	99.61
2.9	99.63	99.64	99.65	99.66	99.67	99.68	99.69	99.70	99.71	99.72
3.0	99.73									
3.5	99.95									
4.0	99.994									
4.5	99.9993									
5.0	99.99994									

Table III. t Distribution ($t_{\alpha,\nu}$)

ν \ α	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

ν = degrees of freedom.

Appendix B. Summary of Single-Sample Hypothesis Testing

Null Hypothesis	Test Statistic	Alternative Hypothesis	P-Value	Criteria for Rejection, Fixed-Level Test
$H_0: \mu = \mu_0$ σ^2 known	$z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$H_1: \mu \neq \mu_0$	$2[1 - \Phi(\lvert z_0 \rvert)]$	$\lvert z_0 \rvert > z_{\alpha/2}$
		$H_1: \mu > \mu_0$	$1 - \Phi(z_0)$	$z_0 > z_\alpha$
		$H_1: \mu < \mu_0$	$\Phi(z_0)$	$z_0 < -z_\alpha$
$H_0: \mu = \mu_0$ σ^2 unknown	$t_0 = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$H_1: \mu \neq \mu_0$	Probability above t_0 plus probability below $-t_0$	$\lvert t_0 \rvert > t_{\alpha/2,n-1}$
		$H_1: \mu > \mu_0$	Probability above t_0	$t_0 > t_{\alpha,n-1}$
		$H_1: \mu < \mu_0$	Probability below t_0	$t_0 < -t_{\alpha,n-1}$
$H_0: \sigma^2 = \sigma_0^2$	$\chi_0^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$	$H_1: \sigma^2 \neq \sigma_0^2$	2 (Probability beyond χ_0^2)	$\chi_0^2 > \chi^2_{\alpha/2,n-1}$ or $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$
		$H_1: \sigma^2 > \sigma_0^2$	Probability above χ_0^2	$\chi_0^2 > \chi^2_{\alpha,n-1}$
		$H_1: \sigma^2 < \sigma_0^2$	Probability below χ_0^2	$\chi_0^2 < \chi^2_{1-\alpha,n-1}$
$H_0: p = p_0$	$z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$	$H_1: p \neq p_0$	$2[1 - \Phi(\lvert z_0 \rvert)]$	$\lvert z_0 \rvert > z_{\alpha/2}$
		$H_1: p > p_0$	$1 - \Phi(z_0)$	$z_0 > z_\alpha$
		$H_1: p < p_0$	$\Phi(z_0)$	$z_0 < -z_\alpha$

Problem Type	Point Estimate	Type of Interval	$100(1-\alpha)\%$ Confidence Interval
Confidence interval on the mean μ, variance σ^2 known	$\bar{x}$	Two-sided	$\bar{x} - z_{\alpha/2}\sigma/\sqrt{n} \leq \mu \leq \bar{x} + z_{\alpha/2}\sigma/\sqrt{n}$
		One-sided lower	$\bar{x} - z_\alpha\sigma/\sqrt{n} \leq \mu$
		One-sided upper	$\mu \leq \bar{x} + z_\alpha\sigma/\sqrt{n}$
Confidence interval on the mean μ of a normal distribution, variance σ^2 unknown	$\bar{x}$	Two-sided	$\bar{x} - t_{\alpha/2,n-1}s/\sqrt{n} \leq \mu \leq \bar{x} + t_{\alpha/2,n-1}s/\sqrt{n}$
		One-sided lower	$\bar{x} - t_{\alpha,n-1}s/\sqrt{n} \leq \mu$
		One-sided upper	$\mu \leq \bar{x} + t_{\alpha,n-1}s/\sqrt{n}$
Confidence interval on the variance σ^2 of a normal distribution	s^2	Two-sided	$\dfrac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} \leq \sigma^2 \leq \dfrac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}$
		One-sided lower	$\dfrac{(n-1)s^2}{\chi^2_{\alpha,n-1}} \leq \sigma^2$
		One-sided upper	$\sigma^2 \leq \dfrac{(n-1)s^2}{\chi^2_{1-\alpha,n-1}}$
Confidence interval on a proportion or parameter of a binomial distribution p	$\hat{p}$	Two-sided	$\hat{p} - z_{\alpha/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{\alpha/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$
		One-sided lower	$\hat{p} - z_\alpha\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}} \leq p$
		One-sided upper	$p \leq \hat{p} + z_\alpha\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$